STELLAR VELOCITY DISTRIBUTION

AS DERIVED FROM OBSERVATIONS IN THE LINE OF SIGHT

BY

WALTER GYLLENBERG
LIC. PHIL. MLM.

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WITH 18 FIGURES IN TEXT

ACAD.
LUGD BAT.
BIBL.

LUND
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LEIPZIG
OTTO HARRASSOWITZ

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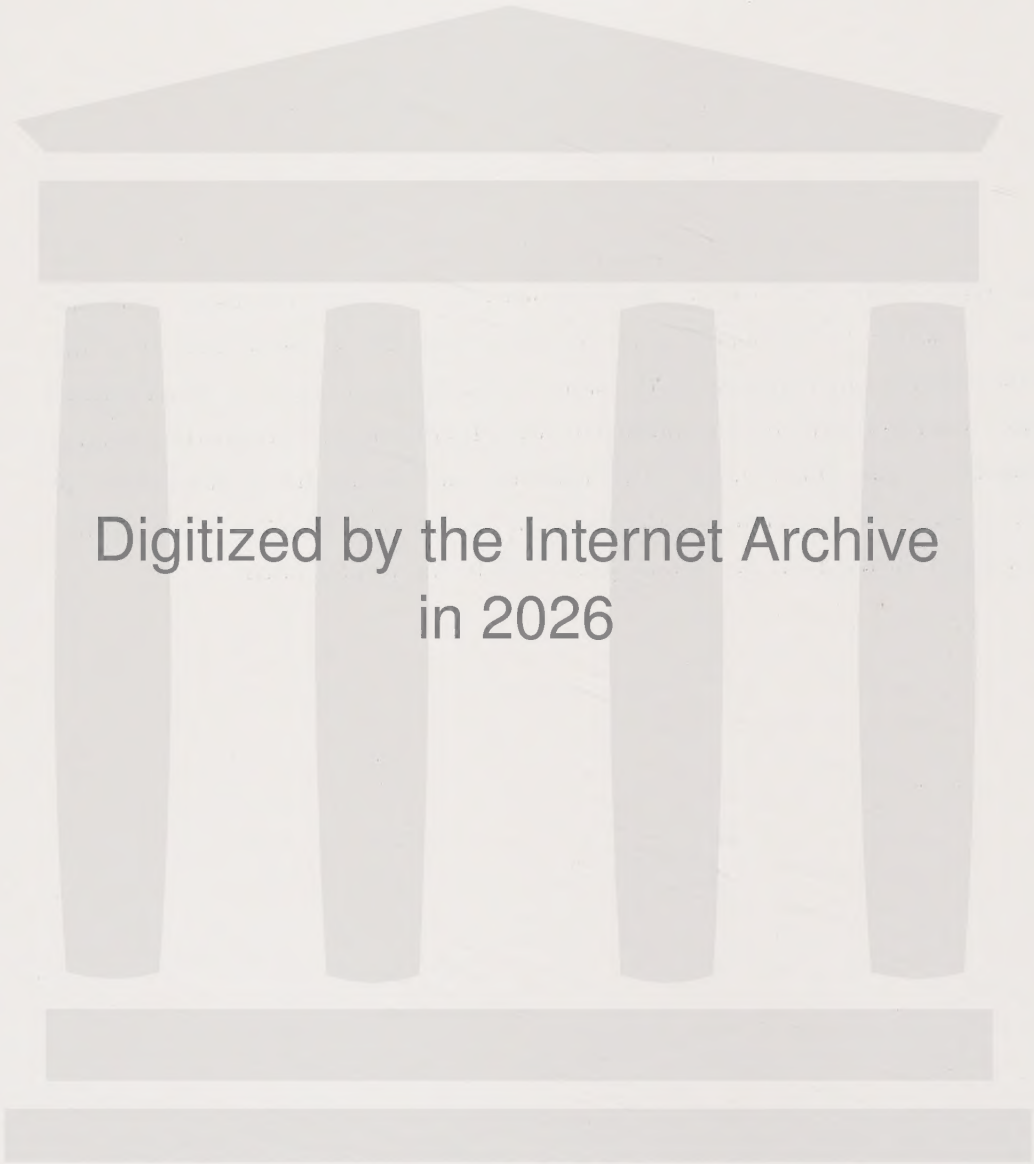
In the years 1911—1913 Prof. Charlier delivered a series of lectures on *Stellar Statistics* of which an account is given in *Meddelande* No. 8 and 9 serie II of the *Lund Observatory*. In accession to the work of Charlier a series of investigations on *Stellar Statistics* have been published in the *Meddelande* series I and II by several astronomers of the Observatory. The following memoir is one of this series. It contains a statistical research on the radial velocities for which the characteristics of the velocity distribution are given principally to the second order.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CHAPTER I.

Historical Introduction.

1. The history of the radial velocity determinations of the stars is — as far as the last century is concerned — very short. The results deduced from the material, so laboriously brought together, have also been very uncertain. The most interesting problem, to determine the motion of the sun relative to the stars, from observations in the line of sight, led to very varying values concerning the magnitude of the velocity as well as its direction. In fact, as late as 1900 these fundamental quantities were known from radial velocity stars with very small accuracy.

Undoubtedly there were many difficulties to overcome especially regarding the small sensibility of the instruments, but another and perhaps greater difficulty was to be found in the circumstance that the measurements of the displacement of the spectral lines were made visually. Evidently in this way it was only possible to measure the spectra of the brightest stars.

In 1875 and the following years systematic radial velocity determinations were made at the *Greenwich Observatory* especially by Mr. MAUNDER. These results, published in the *Monthly Notices* [1], were subjected to discussion by different persons and many attempts were made to determine the velocity of the sun in space.

In 1883 KÖVESLIGETHY [2] makes, perhaps, the first determination of the solar velocity. The number of stars, however, being too small to give a good value of the direction of the apex, he assumes this latter to be known. Adopting the values of the coordinates of the apex derived from proper motion observations $\alpha = 261^0.0 \quad \delta = +35^0.1$ he got the mean value of the velocities of the stars surrounding this apex to be 63.8 km per sec. The mean value of the velocities of the stars surrounding the parallactic equator was found to be 7.5 km per sec. and he considered this evidence as proof of the accuracy of the solution.

Some years later, in 1885 HOMANN [3] makes a more complete solution, based upon different sets of spectroscopic observations, namely the measurements previously

[1] Monthly Notices Tome XXXII and following.

[2] Astron. Nachr. **114**. 327. 1886.

[3] In Astron. Nachr. **114**. 25. 1886 the author gives a short summary of his results.

mentioned made at *Greenwich*, and besides observations made by HUGGINS and SEABROKE. The results obtained by HOMANN from these observations were the following:

Observations	Apex		Sun's relative velocity
	α	δ	
Greenwich	$320^0.1$	$+ 41^0.2$	39.3 ± 4.2 km,
HUGGIN'S	$309^0.5$	$+ 69^0.7$	48.5 ± 23.1 » ,
SEABROKE'S	$278^0.8$	$+ 13^0.6$	24.5 ± 15.8 » .

Mr. HOMANN was very well satisfied with these elements, and the values obtained from the latter two series, seem to him in spite of the small number of stars entered to agree very well with the results from the *Greenwich* observations. He says also in the report of his paper, that in spite of the large difference in the results, a certain agreement may be observed, and especially concerning the relative velocity of the sun we may infer that this can scarcely diverge to a any considerable extent from 30 km per sec.

2. In 1888 the first systematic attempts to measure the displacements of the spectral lines with aid of photographic methods were made by H. C. VOGEL. In this way it was possible to get a considerably greater exactness of the results. In the course of the next four years he measures the spectra of 51 of the brightest stars [1] and assumes that the mean error in every measurement does not come up to 2.0 km per sec. That the number of stars was so small was exclusively due to the instrument used.

From these observations KEMPF [2] and RISTEEN [3] tried to determine the velocity of the sun and the solar apex. The following results were obtained by KEMPF:

Position of the apex $\alpha = 206^0.1 \pm 12^0.0$, $\delta = + 45^0.2 \pm 9^0.2$,

Sun's relative velocity $S = 18.6 \pm 2.9$ km per sec.

In his solution, each star was entered with an individual weight. Another solution, where all stars were assumed to have the same weight, gave no principal deviation from the above results. It seemed, however, to him very important to make a new solution where neighbouring stars, obviously belonging to the same cluster, were rejected, all except one. The number of stars in this way diminished to 41 led to the following solution:

Position of the apex $\alpha = 159^0.7 \pm 20^0.2$, $\delta = + 50^0.2 \pm 14^0.3$,

Sun's relative velocity $S = 13.0 \pm 3.3$ km per sec.

[1] Publ. des Astrophysikalischen Observatoriums zu Potsdam. VII: 1. 1892.

[2] Versuch einer Ableitung der Bewegung des Sonnensystems aus den Potsdamer Spectrographischen Beobachtungen von H. C. VOGEL. Astr. Nachr. **132**. 81. 1893.

[3] The sun's motion through space, by A. D. RISTEEN. The Astronom. Journal. **13**. 75. 1893.

Like KEMPF, RISTEEN makes an exclusion of stars apparently belonging to the same clusters and pufs in their place only one star. From the remaining 42 stars he gets elements of the solar motion that do not in any high degree differ from the results of KEMPF. His value of the relative velocity of the sun is 17.5 km per sec.

3. The successful measurement of stellar radial velocities at the *Potsdam Observatory* with the aid of photographic methods soon led to many attempts on the same subject by other observatories, and it was in the first place at the *Lick* and the *Yerkes* Observatories that the measurements of stellar spectra were put on the working programs. The sensibility of the instruments of these observatories made it not only possible to get radial velocity determinations of fainter stars but also gave the measurements a precision that was much greater than could be obtained at other observatories.

It was chiefly at the *Lick Observatory,* under the direction of Prof. W. W. CAMPBELL, that the radial velocity measurements were persued and in the year 1901 CAMPBELL had at his disposal 280 of the brightest stars mainly of north declination whose velocities in the line of sight are well determined.

In a paper published in *Astrophysical Journal* [1] he gives the following results for the solar motion:

Position of the apex: $\alpha = 277^{0}\,30' \pm 4^{0}.8$, $\delta = +19^{0}\,58' \pm 5^{0}.9$,

The relative velocity of the sun: 19.89 ± 1.52 km per sec.

These results are in good agreement with those later obtained. CAMPBELL makes, however, a further examination of his observations. The values for the apex direction and the solar velocity being known, it is possible to make a special examination of the peculiar velocities of the stars in the line of sight. To this end he corrects every observed radial velocity for the influence of the motion of the sun, and of the thus remaining 280 peculiar velocities he determines the average value without regarding the signs. This average he finds to be 17.05 km per sec. To get an idea of the influence of spectral type upon average velocity he makes a rough separation of the stars in two groups approximatively representing white and red stars. The difference, however, in the average velocities of the two groups seems hardly sufficient to justify any statement as to the effect of spectral type upon velocity.

There seemed, on the contrary, to be a larger correlation between the visual brightness and the velocity, and he says that the progression is in no way due to an increase of probable error of the velocity determination with decreasing magnitude, but must certainly have a real basis. As was later shown, however, these results were exclusively due to the definition of the spectral types and the distribution of the spectral types within the different stellar magnitudes.

[1] A preliminary determination of the motion of the stars by W. W. CAMPBELL. Astr. Phys. Journal, XIII. p. 80.

4. In the following years the systematic work on radial velocity determinations was continued, especially at the *Lick*, *Yerkes*, *Allegheny* and *Mount Wilson* Observatories. In many cases, where the velocity, due to a binary character, was found to be variable, the velocity of the centre of mass was determined.

In the year 1911, in two papers, »On the motion of the brighter class *A* stars» [1] and »Some peculiarities in the motion of the stars» [2] CAMPBELL makes a complete summary of the results up to that time drawn from the radial velocities of the stars. In the first paper he discusses the motions of 225 class *B* stars mainly brighter than magn. 5.0 whose velocities in the line of sight are determined. At the same time he publishes the individual observations. From these stars he derives the position of the apex and the relative velocity of the sun. Moreover he makes here the discovery of the remarkable constant *K* the explanation of which seems to him to be found in certain atmospheric conditions of the stars of this class. In the same paper he points out the very small avarage velocity of the class *B* stars. FROST and ADAMS had however as early as 1904 called attention to the small peculiar velocities of these stars.

Observing the dependence of spectral classes upon radial peculiar velocity, he makes a new investigation of the correlation between visual magnitude, spectral type and velocity. Dividing the stars into two spectral classes, SECCHI's types I and II, he gets the following table, reprinted from his paper.

Average velocities.

Vis. Mag.	Type I		Type II		Types I and II	
	No.	*V* km.	No.	*V* km.	No.	*V* km.
Above 1.50	9	10.5	8	16.8	17	12.2
1.51 to 2.50	26	9.2	22	12.3	48	10.7
2.51 to 3.50	40	9.4	74	15.0	114	13.0
3.51 to 4.50	116	8.9	239	15.1	355	13.0
4.51 to 5.50	126	11.3	336	15.1	462	14.1
5.51 to 6.50	9	19.1	18	18.5	27	18.7
Below 6.50	4	11.3	8	12.5	12	12.1
Means	330	10.25	704	15.08	1034	13.51

The influence of spectral class upon velocity was shown more clearly when the classification of the stars was made in greater detail. The strong correlation between magnitude and velocity is here quite obliterated.

In order to test the two-drift theory of KAPTEYN, CAMPBELL computes the average residual velocity in zones at different distances from the vertex found by KAPTEYN $\alpha = 93^0$ $\delta = +12^0$. Concerning the class *B* stars the difference of the average speed in the zones seems to give no evidence for the existence of a vertex.

[1] Lick. Obs. Bull. VI. **195**. p. 101.
[2] Lick. Obs. Bull. VI. **196**. p. 125.

In another paper[1], however, the same examination was made for the class *A* stars. Here a certain variation of the average peculiar velocity was noted not only in terms of different distances from the assumed vertex but also in terms of galactic latitudes. The vertex being situated in the plane of the Milky Way, this latter variation is however quite evidently due to the former. He says about these fact that »there seems to be no doubt, at least so far as the stars thus far observed are concerned, that Class *B* radial velocities do not increase as their angular distances from KAPTEYN's vertices decrease, but that class *A* velocities do show increase; in fact the preferential-motion effect is apparently stronger for class *A* stars than for stars of classes *F*, *G*, *K* and *M*».

The radial velocity observations made at the *Lick* observatory during 1900 and the following years were not totally published until 1913, and consequently only a few authors have worked on this subject. The results obtained up to that time from radial velocity results are clearly outlined by CAMPBELL in his recently issued book *Stellar Motions*.

Catalogue of the observations of the velocities of the stars in the line of sight.

5. The catalogue of published radial velocities, which I have brought together, is collected principally from the published records of the *Lick* and the *Allegheny* observatories. Most of the data are indeed obtained from the former observatory and especially from the list of 915 stars with known radial velocities published by CAMPBELL in the *Lick Observatory Bulletin T. VII No. 229*. In all, the catalogue contains the radial motion of more than 1500 stars.

The present investigation is consequently based on material containing in the first instance modern information from the catalogue mentioned. Further the lists of observations found in earlier publications of the *Lick* and *Allegheny* observatories are used, and with the help of publications of other observatories the material is increased with such stars of which no records are to be found in the mentioned Bulletins. Such stars are observed at the observatories of *Bonn* and *Mount Wilson* and are published in the *Astronomische Nachrichten* or the *Astrophysical Journal*.

In some of the lists[2], last referred to, the corrections to the »Lick system» are also given. In such cases I have used the reduced values in the catalogue.

6. By collecting the observations it was found that a fairly large number of stars — between two or three hundred — have variable radial velocity. In the cases when the stars are contained in CAMPBELL's catalogues of spectroscopic binary stars I have adopted the velocity of the centre of mass of the system, given there.

[1] Preliminary radial velocities of 212 brighter class A stars. Lick. Obs. Bull. VII. **211**. p. 19.

[2] See F. KÜSTNER: Radialgeschwindigkeiten von 227 Sternen des Spectraltypus F bis M, beobachtet 1908 bis 1913 am Bonner 30 cm Refractor. Astr. Nachr. Bd 198 No. 4750.

In other cases I have hesitated about entering observations of this kind in the catalogue, in particular as in many cases the observations show such an irregularity that the estimation of the velocity of the centre of mass is quite hopeless.

For several of those stars with variable radial velocity the observations are however numerous and so arranged that the amplitude comes forth with sufficient clearness to give an acceptable value of the velocity of the binary system. In other cases the stars have so small an amplitude, that a direct determination of the velocity of the centre of mass will be affected only by a small error. The velocities were for these stars obtained by taking a mean between the highest and lowest velocity observed.

7. The stars for which in this manner it is possible, though with some uncertainty, to obtain a value of the velocity of the centre of mass are all entered in the catalogue. For all those stars I have however marked out the variation in velocity and the uncertainty in the determination. There are in all 65 stars of types *B* and *A*, having roughly determined velocities. These stars are all included in the determination of the apex, but when the second moments are computed I have rejected them, because their large errors may too much influence the results.

Doubtless there are amongst the stars noted to have constant velocities many that are in reality variables and amongst them many which have probably very large or very short periods. The observations obtainable are all made during the last ten years. Most of them are observed during a very short time and for several stars the velocities are computed from only a small number of observations.

The catalogue contains no nebulæ, but 5 stars of the spectral type *Oe* are included.

For each star a card was prepared containing: the name or signification of the star, its coordinates for 1900.0, its radial velocity and the proper motions in right ascension and declination, the latter being taken from *Boss' Preliminary catalogue*. For the stars not found in this catalogue the proper motions were taken from other sources.

On the cards the spectral types and the magnitudes taken from the *Revised Harvard Photometry Annals, H. C. O. No. 50* were also noted.

The velocity in right ascension and declination are expressed in seconds of arc, but concerning the velocity in the line of sight, I have given up the usual unity — km per sec. — and accepted unities more convenient in Stellar Astronomy, Siriometer and Stellar year. These units, introduced by Charlier in his lectures, have the following definitions:

One Siriometer (Sm) $= 10^6$ the mean distance of the earth from the sun.
One Stellar year (St) $= 10^6$ usual years.

The relations for transforming into the new units are the following:

1 km per sec. $= 0.2111$ Sm per St.
1 Sm per St. $= 4.7375$ km per sec.

The catalogue contains several stars, whose parallaxes are determined. Most of these stars were found in a paper by Walter S. Adams and Arnold Kohlschütter: *The radial velocities of one hundred stars with measured parallaxes* [1]. These stars I have separately examined in the cases when the parallax permitted a good determination of the absolute velocity. From 160 such stars with $\pi \geq 0''.025$ I have deduced the sun's relative motion and, applying the generalized ellipsoidal hypothesis, given an approximative aspect of the velocity distribution [2].

The total number of stars collected are in all 1640.

The stars are chiefly of bright magnitudes and the catalogue may broadly be considered complete down to the magnitude 5.0. The motions in the line of sight being however independent of the distances of the stars, nothing prevents us from treating stars of different magnitudes, when the spectral types are separated. It is however necessary, by summing up the spectral types, to limit the magnitude in order to get results comparable with those found from proper motions.

[1] Contribution from the Mount Wilson Solar Observatory. No. **78**, **79**.

[2] Meddelande fr. Lunds Astr. Obs. No. **65**.

CHAPTER II.

The different systems of coordinates to which the velocities are referred.

8. In the present investigation all velocities and results are referred to right-angular Cartesian systems of coordinates. In this paragraph these will be defined. The denominations of the systems as well as the direction cosines of their axes are taken from the memoir of CHARLIER [1].

The following three systems of coordinates are used:

The system K_1, having its three axes coincident with those of the velocity ellipsoid, the positive X-axis directed towards the principal vertex [2]. This system will be further defined in connection with the ellipsoidal hypothesis.

The system K_2 is the usual astronomical system of coordinates. The $X\ Y$ plane coincides with equator, the positive X-axis is directed toward the equinoctium and the positive Z-axis towards the north pole.

The system K_3 is determined through the position of each square. It has its positive Z-axis directed towards the centre of gravity of the square. The positive X and Y axes are directed towards increasing right ascension and declination.

This latter system of coordinates is not quite identical with the system K_3 used by CHARLIER in the cited memoir. There he assumes the positive X and Y axes to be directed towards *decreasing* right ascension and declination.

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[1] Meddelande fr. Lunds Astr. Obs. Serie II. No. 9.

[2] I will in the following use the indication *principal vertex* for the point corresponding to the antivertex of KAPTEYN. The addendum *principal* is necessary as the velocity distribution is expressed through a three axial ellipsoid.

The following schemes now give the denominations of the systems as well as the direction cosines between their axes.

		K_2:		
		X''	Y''	Z''
K_3:	X	γ_{11}	γ_{21}	γ_{31}
	Y	γ_{12}	γ_{22}	γ_{32}
	Z	γ_{13}	γ_{23}	γ_{33}

		K_2:		
		X''	Y''	Z''
K_1:	X'	ε_{11}	ε_{21}	ε_{31}
	Y'	ε_{12}	ε_{22}	ε_{32}
	Z'	ε_{13}	ε_{23}	ε_{33}

For the rest I refer to the memoir of CHARLIER.

9. The new directions of the X and Y axes will naturally only change the corresponding direction cosines regarding their signs. I will however reprint these direction cosines of K_3 relative K_2 after the correction (see table I).

Determination of the direction and the magnitude of the relative motion of the sun from radial velocity observations.

10. There are, as well known, amongst the radial velocity determinations several stars whose velocities are exceptionally large. These are chiefly to be found amongst the so called later types. The presence of a large number of such stars may perhaps partly be explained by the circumstances, which make it possible to get good values by measuring the displacements of the spectral lines on the photographic plates. Particularly amongst the faint stars, a large displacement is more easy to discover and determine than a small one, and moreover for stars of types, whose spectra contain a large number of lines, the velocity determination will be more easy than for stars of other types.

It is, however, necessary in a statistical investigation like this to reject very large velocities, while their influence by the method of moments will be too great. I have, however, striven to reject as few stars as possible. From a preliminary examination of the velocity distribution, I found it convenient to reject those stars, as whose velocities exceed 14 Siriometer per Stellar year or 66.3 km per second.

The stars in such way rejected seem chiefly to belong to the fainter magnitudes. A brief computation gives the following distribution of the large velocities.

Rad. velocity	Number of stars magn. ≤ 4.9	Number of stars magn. ≥ 5.0	Total number
$\rho \leq 14.0$ Sm/St.	1115	481	1596
$\rho \geq 14.0$ »	16	28	44
Stars with large velocity in per cent	1.41	5.50	2.68

TABLE I.

Direction cosines for the different squares.

	γ_{11}	γ_{12}	γ_{13}	γ_{21}	γ_{22}	γ_{23}	γ_{32}	γ_{33}
A_1	— 1.0000	0.0000	0.0000	0.0000	— 0.9848	+ 0.1736	+ 0.1736	+ 0.9848
A_2	+ 1.0000	0.0000	0.0000	0.0000	+ 0.9848	— 0.1736	+ 0.1736	+ 0.9848
B_1	— 0.3090	— 0.6737	+ 0 6714	+ 0.9511	— 0.2189	+ 0.2181	+ 0.7059	+ 0.7083
B_2	— 0.8090	— 0.4163	+ 0.4149	+ 0.5878	— 0.5730	+ 0.5711	+ 0.7059	+ 0.7083
B_3	— 1 0000	0.0000	0.0000	0.0000	— 0.7083	+ 0.7059	+ 0.7059	+ 0.7083
B_4	— 0.8090	+ 0.4163	— 0.4149	— 0.5878	— 0.5730	+ 0.5711	+ 0.7059	+ 0.7083
B_5	— 0.3090	+ 0.6737	— 0.6714	— 0.9511	— 0.2189	+ 0.2181	+ 0.7059	+ 0.7083
B_6	+ 0.3090	+ 0.6737	— 0.6714	— 0.9511	+ 0.2189	— 0 2181	+ 0.7059	+ 0.7083
B_7	+ 0.8090	+ 0.4163	— 0.4149	— 0.5878	+ 0.5730	— 0.5711	+ 0.7059	+ 0.7083
B_8	+ 1.0000	0.0000	0.0000	0.0000	+ 0.7083	— 0.7059	+ 0.7059	+ 0.7083
B_9	+ 0.8090	— 0.4163	+ 0.4149	+ 0.5878	+ 0.5730	— 0.5711	+ 0.7059	+ 0.7083
B_{10}	+ 0.3090	— 0.6737	+ 0.6714	+ 0.9511	+ 0.2189	— 0 2181	+ 0.7059	+ 0.7083
C_1	— 0.2588	— 0.2415	+ 0.9353	+ 0.9659	— 0.0647	+ 0.2506	+ 0.9683	+ 0.2500
C_2	— 0.7071	— 0.1768	+ 0.6847	+ 0.7071	— 0.1768	+ 0.6847	+ 0.9683	+ 0.2500
C_3	— 0.9659	— 0.0647	+ 0 2506	+ 0.2588	— 0.2415	+ 0.9353	+ 0.9683	+ 0.2500
C_4	— 0.9659	+ 0.0647	— 0.2506	— 0.2588	— 0.2415	+ 0.9353	+ 0.9683	+ 0.2500
C_5	— 0.7071	+ 0.1768	— 0.6847	— 0.7071	— 0.1768	+ 0.6847	+ 0.9683	+ 0.2500
C_6	— 0.2588	+ 0.2415	— 0.9353	— 0.9659	— 0 0647	+ 0.2506	+ 0.9683	+ 0 2500
C_7	+ 0.2588	+ 0.2415	— 0.9353	— 0.9659	+ 0.0647	— 0.2506	+ 0.9683	+ 0.2500
C_8	+ 0.7071	+ 0.1768	— 0.6847	— 0.7071	+ 0.1768	— 0.6847	+ 0.9683	+ 0.2500
C_9	+ 0.9659	+ 0.0647	— 0.2506	— 0.2588	+ 0.2415	— 0.9353	+ 0.9683	+ 0.2500
C_{10}	+ 0.9659	— 0.0647	+ 0.2506	+ 0.2588	+ 0.2415	— 0.9353	+ 0.9683	+ 0.2500
C_{11}	+ 0.7071	— 0.1768	+ 0.6847	+ 0.7071	+ 0.1768	— 0.6847	+ 0.9683	+ 0.2500
C_{12}	+ 0.2588	— 0.2415	+ 0.9353	+ 0.9659	+ 0.0647	— 0.2506	+ 0.9683	+ 0.2500
D_1	— 0.2588	+ 0.2415	+ 0.9353	+ 0.9659	+ 0.0647	+ 0.2506	+ 0.9683	— 0.2500
D_2	— 0.7071	+ 0.1768	+ 0.6847	+ 0.7071	+ 0.1768	+ 0.6847	+ 0.9683	— 0.2500
D_3	— 0.9659	+ 0.0647	+ 0.2506	+ 0.2588	+ 0.2415	+ 0.9353	+ 0.9683	— 0 2500
D_4	— 0.9659	— 0.0647	— 0.2506	— 0.2588	+ 0.2415	+ 0.9353	+ 0 9683	— 0 2500
D_5	— 0.7071	— 0.1768	— 0.6847	— 0.7071	+ 0.1768	+ 0.6847	+ 0.9683	— 0.2500
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D_{11}	+ 0.7071	+ 0 1768	+ 0 6847	+ 0.7071	— 0.1768	— 0.6847	+ 0.9683	— 0.2500
D_{12}	+ 0.2588	+ 0.2415	+ 0.9353	+ 0.9659	— 0.0647	— 0.2506	+ 0.9683	— 0.2500
E_1	— 0.3090	+ 0.6737	+ 0.6714	+ 0.9511	+ 0 2189	+ 0.2181	+ 0.7059	— 0.7083
E_2	— 0.8090	+ 0.4163	+ 0.4149	+ 0.5878	+ 0.5730	+ 0.5711	+ 0.7059	— 0.7083
E_3	— 1.0000	0.0000	0.0000	0.0000	+ 0.7083	+ 0.7059	+ 0.7059	— 0.7083
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E_6	+ 0.3090	— 0 6737	— 0.6714	— 0.9511	— 0.2189	— 0.2181	+ 0 7059	— 0.7083
E_7	+ 0.8090	— 0.4163	— 0.4149	— 0.5878	— 0.5730	— 0.5711	+ 0.7059	— 0.7083
E_8	+ 1.0000	0.0000	0.0000	0.0000	— 0.7083	— 0.7059	+ 0.7059	— 0.7083
E_9	+ 0.8090	+ 0.4163	+ 0 4149	+ 0.5878	— 0.5730	— 0.5711	+ 0.7059	— 0.7083
E_{10}	+ 0.3090	+ 0.6737	+ 0.6714	+ 0.9511	— 0.2189	— 0.2181	+ 0.7059	— 0.7083
F_1	— 1.0000	0.0000	0.0000	0.0000	+ 0.9848	+ 0.1736	+ 0.1736	— 0.9848
F_2	+ 1.0000	0.0000	0.0000	0.0000	— 0.9848	— 0.1736	+ 0.1736	— 0.9848

From this comparison we see that the large velocities are mainly to be found amongst the fainter stars. On the other side these stars are exclusively of so called later types.

11. The following table shows the distribution of the remaining stars over the sky. The second column contains the number of stars, whose velocities do not come up to 14 Sm per St. The other columns contain the number of rejected stars (see table II).

In each square the stars were now distributed according to their spectral classes and magnitudes, brighter and fainter than 5.0. For every group the mean displacement was computed.

The table III gives these results. The table contains the number of stars, the mean displacements in each square and finally the number of rejected stars. The means are directly computed. In the two groups, »magnitude ≤ 4.9» and »all stars», five *Oe* class stars are added.

12. From these means we now go to determine the solar apex and related constants.

TABLE II.

Distribution over the sky of stars, whose velocities in the line of sight are known.

Square	n	$z \geqq 14.0$	Square	n	$z \geqq 14.0$
A_1	18	—	D_1	17	
A_2	30	1	D_2	24	—
			D_3	41	2
B_1	42	2	D_4	39	2
B_2	36	3	D_5	24	—
B_3	28	1	D_6	12	—
B_4	27	1	D_7	15	—
B_5	30	2	D_8	25	3
B_6	24	—	D_9	37	—
B_7	24	3	D_{10}	31	—
B_8	35	1	D_{11}	31	—
B_9	42	2	D_{12}	19	—
B_{10}	33	1			
			E_1	36	2
C_1	24	1	E_2	38	1
C_2	29	2	E_3	45	2
C_3	55	2	E_4	69	2
C_4	34	1	E_5	55	—
C_5	25	—	E_6	51	—
C_6	23	—	E_7	52	1
C_7	35	—	E_8	46	1
C_8	31	—	E_9	36	1
C_9	25	—	E_{10}	35	—
C_{10}	40	2			
C_{11}	29	1	F_1	40	—
C_{12}	30	—	F_2	29	1

Let us assume that the velocity of the sun relative to the stars is S Siriometer, and let this velocity projected in the usual astronomical system of coordinates — the system K_2 — have the components

$$U_0'', \quad V_0'', \quad W_0''$$

along the X'', Y'' and Z'' axes, so that

$$S^2 = U_0''^2 + V_0''^2 + W_0''^2. \tag{1}$$

Assuming further the component of the same velocity in a certain square, or with other words in the K_3-system to have the value ε_0 along the Z axis, the scheme page 13 gives this latter component expressed through the former by the relation

$$\gamma_{13} U_0'' + \gamma_{23} V_0'' + \gamma_{33} W_0'' = \varepsilon_0. \tag{2}$$

This relation is the very equation we have to solve with respect to the unknown quantities

$$U_0'', \quad V_0'', \quad W_0''.$$

TABLE III.

The number of stars in the squares of each spectral group and their mean displacement.

Square	Spectral classes											
	B			*A*			*F*			*G*		
	n	z_0	rejected stars	n	z_0	rejected stars	n	z_0	rejected stars	n	z_0	rejected stars
A_1	—	—	—	4	— 2.54	—	4	— 1.34	—	2	— 1.30	—
A_2	2	— 2.74	—	2	— 1.38	—	5	— 0.55	—	6	— 2.38	—
B_1	11	— 0.91	—	7	+ 0.37	—	5	+ 2.55	1	6	— 2.10	1
B_2	11	+ 0.83	—	1	+ 5.28	1	6	+ 1.49	—	9	+ 1.73	2
B_3	2	+ 1.90	—	8	+ 0.97	—	2	+ 0.42	1	4	+ 6.17	—
B_4	1	+ 3.38	—	8	+ 0.59	—	6	+ 1.20	1	6	+ 3.02	—
B_5	—	—	—	9	— 1.43	—	8	— 0.43	—	4	+ 1.09	1
B_6	1	— 1.27	—	11	— 2.05	—	4	— 1.08	—	3	— 1.45	—
B_7	2	— 0.94	—	3	— 1.63	—	6	— 4.60	—	6	— 5.63	2
B_8	6	— 3.83	—	7	— 4.09	—	7	— 3.21	—	5	— 8.02	1
B_9	9	— 2.94	—	5	— 2.34	—	8	— 4.11	—	5	— 3.36	1
B_{10}	9	— 3.73	—	5	— 0.46	—	3	— 2.56	1	2	— 1.07	—
C_1	2	+ 1.49	—	6	+ 0.36	—	1	— 1.48	—	5	— 0.02	1
C_2	10	+ 1.79	—	5	+ 3.35	1	1	+ 5.43	1	4	+ 0.26	—
C_3	13	+ 3.99	—	18	+ 6.85	1	7	+ 4.11	—	6	+ 4.28	—
C_4	2	+ 8.02	—	6	+ 1.53	—	4	+ 2.87	—	9	+ 0.86	—
C_5	2	+ 4.36	—	7	+ 0.42	—	3	+ 4.20	—	4	+ 2.66	—
C_6	1	+ 9.84	—	9	— 0.86	—	7	+ 0.89	—	1	— 1.90	—
C_7	—		—	13	+ 0.11	—	3	— 5.01	—	7	— 1.38	—
C_8	—	—	—	7	— 2.45	—	5	— 1.88	—	5	— 3.46	—
C_9	1	— 0.84	—	7	— 2.40	—	3	— 9.57	—	4	— 2.12	—
C_{10}	5	— 2.33	—	9	— 5.53	—	4	— 1.81	—	8	— 1.97	1
C_{11}	3	— 3.38	—	6	— 2.50	—	8	— 2.15	—	5	— 4.54	—
C_{12}	3	+ 1.29	—	6	— 0.59	—	6	— 0.13	—	3	— 1.45	—
D_1	1	+ 2.32	—	—	—	—	2	+ 1.22	—	1	+ 4.33	—
D_2	4	+ 2.32	—	4	+ 2.96	—	6	+ 0.67	—	1	+ 5.70	—
D_3	18	+ 5.53	—	5	+ 2.26	—	5	+ 1.44	—	2	— 5.84	1
D_4	15	+ 6.59	1	5	+ 2.61	—	5	+ 6.85	—	3	+ 2.66	—
D_5	1	+ 5.07	—	3	+ 1.48	—	3	+ 5.65	—	8	+ 2.84	—
D_6	1	+ 2.96	—	3	+ 2.75	—	—	—	—	1	— 0.87	—
D_7	2	— 0.57	—	2	+ 0.30	—	3	— 0.90	—	2	— 1.34	—
D_8	3	— 0.97	—	5	— 3.17	1	4	— 1.02	—	—	—	1
D_9	6	— 0.58	—	8	— 3.38	—	11	— 2.21	—	3	— 0.88	—
D_{10}	6	— 1.34	—	2	— 4.11	—	4	— 4.57	—	6	— 2.99	—
D_{11}	2	— 3.54	—	8	— 3.57	—	6	— 0.86	—	7	— 4.46	—
D_{12}	—	-	-	4	+ 1.62	—	1	+ 5.72	—	4	+ 1.24	—
E_1	4	+ 4.12	—	6	+ 1.16	—	8	+ 2.45	—	5	+ 3.52	1
E_2	4	+ 3.72	—	7	+ 2.26	—	7	+ 3.74	—	4	+ 5.17	1
E_3	8	+ 4.59	—	6	+ 5.30	—	6	+ 2.36	—	3	+ 1.97	1
E_4	25	+ 4.99	—	8	+ 3.13	—	8	+ 2.83	1	5	+ 0.98	—
E_5	12	+ 3.13	—	11	+ 0.82	—	8	+ 1.05	—	8	+ 1.86	—
E_6	24	+ 2.40	—	5	— 1.06	—	5	— 2.57	—	3	+ 0.89	
E_7	23	+ 0.24	—	3	— 1.04	—	7	— 2.47	—	4	— 2.81	1
E_8	13	— 0.60	—	6	— 1.43	—	7	— 2.62	—	4	+ 1.56	—
E_9	5	— 0.08	—	7	— 1.00	—	5	— 1.00	—	3	— 1.41	—
E_{10}	2	+ 0.49	—	7	+ 0.96	—	2	— 0.56	—	5	— 0.39	—
F_1	7	+ 3.71	—	4	+ 0.04	—	5	+ 1.89	—	5	+ 2.76	—
F_2	2	+ 2.06	—	3	+ 0.10	—	3	+ 5.99	1	2	+ 2.30	—

TABLE III (continued).

Square	*K*			*M*			magn $\leq$ 4,9			magn $\geq$ 5.0			all stars		
	n	z_0	rejected stars	n	z_0	rejected stars	n	z_0	rejected stars	n	z_0	rejected stars	n	z_0	rejected stars
A_1	6	− 0.16	—	2	+ 1.40	—	10	− 1.535	—	8	− 0.11	—	18	− 0.905	—
A_2	15	− 2.17	1	—	—	—	19	− 1.128	—	11	− 3.31	1	30	− 1.927	1
B_1	10	− 2.86	—	2	+ 0.53	—	26	− 0.379	—	15	− 1.67	2	41	− 0.850	2
B_2	6	+ 0.95	—	3	− 0.10	—	29	+ 1.976	1	7	− 1.86	2	36	+ 1 230	3
B_3	10	+ 0.30	—	2	+ 4.37	—	22	+ 1.731	—	6	+ 1.79	1	28	+ 1.743	1
B_4	6	+ 3.07	—	—	—	—	19	+ 1.766	—	8	+ 2.28	1	27	+ 1.919	1
B_5	8	− 0.90	—	1	− 4.86	1	19	− 0.423	—	11	− 1.46	2	30	− 0.802	2
B_6	2	− 2.64	—	3	− 4.88	—	18	− 2.061	—	6	− 2.55	—	24	− 2.183	—
B_7	5	− 1.12	1	2	− 1.31	—	21	− 2.647	1	3	− 6.41	2	24	− 3.138	3
B_8	8	− 2.61	—	2	− 5.65	—	23	− 4.100	—	12	− 4.32	1	35	− 4.179	1
B_9	14	− 3.97	1	1	− 4.33	—	27	− 3.051	1	15	− 4.36	1	42	− 3.519	2
B_{10}	12	− 2.04	—	2	+ 1.86	—	26	− 2.413	—	8	− 0.53	1	34	− 1.955	1
C_1	9	+ 1.46	—	1	+ 9.48	—	20	+ 1.745	1	4	− 2.19	—	24	+ 1.090	1
C_2	8	+ 1.74	—	1	− 5.80	—	22	+ 0.948	—	7	+ 4.12	2	29	+ 1.714	2
C_3	9	+ 5.95	1	2	+ 4.75	—	43	+ 5.779	2	12	+ 3.69	—	55	+ 5.322	2
C_4	10	+ 3.49	1	2	+ 7.83	—	22	+ 3 437	1	12	+ 1.61	—	34	+ 2.793	1
C_5	8	+ 5.03	—	1	+ 4.92	—	18	+ 4.168	—	7	+ 0.72	—	25	+ 3.202	
C_6	3	− 0.18	—	2	+ 7.09	—	18	+ 0.984	—	5	+ 0.47	—	23	+ 0.872	—
C_7	9	+ 1 71	—	3	− 3.15	—	14	− 1.417	—	21	+ 0.12	—	35	− 0.495	—
C_8	14	− 2 28	—	—	—	—	22	− 2.866	—	9	− 1.41	—	31	− 2.444	—
C_9	9	− 3.94	—	1	− 6.80	—	18	− 4.133	—	7	− 3.38	—	25	− 3.944	—
C_{10}	13	− 4.73	—	1	+ 0.63	1	30	− 4.034	1	10	− 2.43	1	40	− 3.633	2
C_{11}	6	+ 1.31	1	1	− 3.69	—	20	− 1.619	1	9	− 3.17	—	29	− 2.099	1
C_{12}	10	− 0.30	—	2	+ 0.40	—	24	+ 0.224	—	6	− 2.06	—	30	− 0.233	
D_1	11	+ 2.38	—	2	− 0.40	—	13	+ 1.448	—	4	+ 3.91	—	17	+ 2.026	—
D_2	6	+ 2.53	—	3	+10·71	—	16	+ 2.276	—	8	+ 5.21	—	24	+ 3.254	—
D_3	9	+ 3.91	1	1	− 7.11	—	33	+ 3.209	1	8	+ 4.07	1	41	+ 3.377	2
D_4	9	+ 4.50	1	1	+ 8.57	—	32	+ 5.468	2	7	+ 5.19	—	39	+ 5.417	2
D_5	8	+ 5.53	—	1	+ 4.58	—	18	+ 3.612	—	6	+ 5.49	—	24	+ 4.082	—
D_6	6	+ 3.66	—	1	− 2.30	—	10	+ 2.930	—	2	+ 0 83	—	12	+ 2.497	
D_7	4	− 1.75	—	2	+ 3.82	—	15	− 0.351	—	—	—	—	15	− 0.351	—
D_8	12	+ 0.86	1	1	− 0.74	—	21	− 0.696	—	4	+ 0.36	3	25	− 0.527	3
D_9	7	− 1.40	—	2	− 2.38	—	30	− 1.673	—	7	− 3.12	—	37	− 1.946	—
D_{10}	12	− 0.36	—	1	+ 2.87	—	23	− 1.611	—	8	− 2.11	—	31	− 1.740	—
D_{11}	5	− 0 30	—	3	+ 1.68	—	24	− 1.730	—	7	− 3.85	—	31	− 2.208	
D_{12}	8	+ 0.43	—	2	− 2 09	—	17	+ 0.915	—	2	+ 0.41	—	19	+ 0.861	
E_1	11	+ 0.96	1	2	+ 2 37	—	16	+ 1.368	1	20	+ 2.70	1	36	+ 2.109	2
E_2	14	+ 4.54	—	2	− 0.32	—	21	+ 3.733	1	17	+ 3.65	—	38	+ 3.694	1
E_3	18	+ 5.10	1	4	+ 6.97	—	25	+ 3.976	1	20	+ 5.44	1	45	+ 4.626	2
E_4	22	+ 3.43	1	1	+ 4.27	—	47	+ 3.909	1	22	+ 3.14	1	69	+ 3.689	2
E_5	15	+ 3.24	—	1	+ 3.43	—	36	+ 2.487	—	19	+ 2.10	—	55	+ 2.343	—
E_6	11	− 0.25	—	3	+ 4.06	—	44	+ 1.463	—	7	− 1.84	—	51	+ 1.010	—
E_7	13	− 3.20	—	1	− 0.25	—	34	− 1.105	—	18	− 2.52	1	52	− 1.585	1
E_8	14	+ 0.68	1	2	+ 0.74	—	32	− 0.398	—	14	− 0.34	1	46	− 0.381	1
E_9	14	+ 0.85	1	2	+ 3.17	—	17	+ 0.068	—	19	+ 0.02	1	36	+ 0.045	1
E_{10}	15	+ 2.78	—	4	+ 0.47	—	22	+ 1.551	—	13	+ 0.74	—	35	+ 1.258	—
F_1	16	+ 3.35	—	3	+ 1.50	—	22	+ 2.143	—	18	+ 3.35	—	40	+ 2.686	—
F_2	16	+ 1.65	—	3	+ 0.80	—	17	+ 1.497	—	12	+ 2.53	1	29	+ 1.923	1

The same relation must also be valid for each star, if its motion is only due to the velocity of the sun.

If, however, there exists a systematic displacement K, that may be caused by errors in the measurement, by atmospheric conditions of the stars or by a real basis, the equation (2) takes the form

$$\gamma_{13} U_0'' + \gamma_{23} V_0'' + \gamma_{33} W_0'' - \varepsilon_0 + K = 0. \tag{3}$$

The equation thus contains an additional unknown quantity.

13. Consider the following relations:

$$\begin{aligned} \gamma_{13} &= \cos\delta \cos\alpha, \\ \gamma_{23} &= \cos\delta \sin\alpha, \\ \gamma_{33} &= \sin\delta, \end{aligned}$$

where α and δ denote the coordinates of the centre of a square. Assume further A and D to be the angular coordinates of the apex, then we evidently have

$$\begin{aligned} U_0'' &= S \cos D \cos A, \\ V_0'' &= S \cos D \sin A, \\ W_0'' &= S \sin D. \end{aligned} \tag{4}$$

Introducing these values into the equation (2) we find this quite identical with the equation deduced by Campbell [1].

14. Regarding the displacements in the different squares as the observed quantities, we have to solve from 48 equations of condition of the form (2) viz. (3) only three viz. four unknown quantities. Applying the method of least squares we get from (2) the following normal equations:

$$\begin{aligned} [\gamma_{13}\gamma_{13}]\ U_0'' - [\gamma_{13}\varepsilon_0] &= 0, \\ [\gamma_{23}\gamma_{23}]\ V_0'' - [\gamma_{23}\varepsilon_0] &= 0, \\ [\gamma_{33}\gamma_{33}]\ W_0'' - [\gamma_{33}\varepsilon_0] &= 0, \end{aligned} \tag{5}$$

or, when the numerical values of the direction cosines are introduced:

$$\begin{aligned} 16.2344\ U_0'' - [\gamma_{13}\varepsilon_0] &= 0, \\ 16.3552\ V_0'' - [\gamma_{23}\varepsilon_0] &= 0, \\ 15.4132\ W_0'' - [\gamma_{33}\varepsilon_0] &= 0. \end{aligned} \tag{5*}$$

All other coefficients vanish owing to the symmetrical distribution of the squares.

Introducing the fourth unknown quantity K the expressions (5) will not change their form. A new equation independent of the others will be added, namely

$$48K = [\varepsilon_0], \tag{6}$$

where the coefficient of the left membrum is the number of the squares.

[1] Astroph. Journal. XIII. page 80.

As is easily seen, the form (6) involves that all squares have the same weight. Dividing the stars into spectral classes, the number of stars in the different squares is very varying, partly due to the small total number, but also to the real distribution in relation to the Milky Way.

For the solution of the two groups denoted by »magn. ≤ 4.9; stars over the whole sky» and »all stars over the whole sky» the formulæ (5) were used.

When the other groups were treated, account was taken of the weight in each square. Assuming these weights proportional to the numbers of stars, the normal equations now take the form:

$$
\begin{aligned}
&[n\,\gamma_{13}\,\gamma_{13}]\,U_0'' + [n\,\gamma_{13}\,\gamma_{23}]\,V_0'' + [n\,\gamma_{13}\,\gamma_{33}]\,W_0'' + [n\,\gamma_{13}]\,K - [n\,\gamma_{13}\,z_0] = 0,\\
&[n\,\gamma_{23}\,\gamma_{13}]\,U_0'' + [n\,\gamma_{23}\,\gamma_{23}]\,V_0'' + [n\,\gamma_{23}\,\gamma_{33}]\,W_0'' + [n\,\gamma_{23}]\,K - [n\,\gamma_{23}\,z_0] = 0,\\
&[n\,\gamma_{33}\,\gamma_{13}]\,U_0'' + [n\,\gamma_{33}\,\gamma_{23}]\,V_0'' + [n\,\gamma_{33}\,\gamma_{33}]\,W_0'' + [n\,\gamma_{33}]\,K - [n\,\gamma_{33}\,z_0] = 0,\\
&[n\,\gamma_{13}]\quad U_0'' + [n\,\gamma_{23}]\quad V_0'' + [n\,\gamma_{33}]\quad W_0'' + \quad N\quad K - \quad [n\,z_0] = 0.
\end{aligned}
\tag{7}
$$

Here n denotes the number of stars in each square and N the total number of stars. The equation (7) is however nothing but the solution obtained by treating the stars separately. Yet there remains the approximation that all stars in a square are assumed to have the same coordinates as the centre of the square.

15. In tables IV a and IV b the results from these computations are tabulated.

The second column gives the number of stars in each solution. The three following columns give the three components of the sun's relative velocity in the K_2-system. S denotes this velocity and the column K gives the value of this constant for each spectral type.

A new solution was made for each group, assuming the constant K to be zero. These solutions are noted in the last column.

Table IV b contains the results obtained when dividing the stars into magnitudes. In two cases I have made one solution for stars surrounding the pole of the Milky Way and another for stars in the plane of the Milky Way as remarked in the first column.

For this purpose I divided the sky into two equal parts, the one containing 24 squares situated approximatively in a zone with galactic latitudes within $\pm 30^0$, the other part containing the rest of the squares surrounding the pole of the Milky Way. The squares were distributed as follows:

Squares within galactic latitude $\pm 30^0$:

A_1, A_2, B_1, B_2, B_3, B_8, B_9, B_{10}, C_3, C_4, C_{10}, C_{11}, D_4, D_5, D_9, D_{10}, E_3, E_4, E_5, E_6, E_7, E_8, F_1, F_2.

Squares surrounding the pole of the Milky Way:

B_4, B_5, B_6, B_7, C_1, C_2, C_5, C_6, C_7, C_8, C_9, C_{12}, D_1, D_2, D_3, D_6, D_7, D_8, D_{11}, D_{12}, E_1, E_2, E_9, E_{10}.

This division was made to get an idea of the influence of stars in different galactic zones on the apex solution. The results from the two zones seem however

TABLE IV a.

Apex, sun's relative velocity S and the constant K from observations in the line of sight.

Type	Number of stars	U_0''	V_0''	W_0''	S	Apex α	Apex δ	K	Remarks
B-type	284	− 1.175	+ 4.039	− 2.002	4.658	286°.2	+ 25°.5	+ 0.91	individual
»	»	− 0.804	+ 4.239	− 2.705	5.092	280.7	+ 32.1	—	» $K = 0$
A-type	291	+ 0.594	+ 3.990	− 1.044	4.167	261.5	+ 14.5	+ 0.03	»
»	»	+ 0.588	+ 3.986	− 1.041	4.162	261.6	+ 14.5	—	» $K = 0$
F-type	237	+ 0.514	+ 3.890	− 1.224	4.110	262.5	+ 17.3	+ 0.05	»
»	»	+ 0 510	+ 3.885	− 1.226	4.106	262.5	+ 17.4	—	» $K = 0$
G-type	208	+ 0.029	+ 3.624	− 1.595	3.960	279.5	+ 23.8	− 0.17	»
»	»	+ 0.025	+ 3.624	− 1.616	3.969	279.6	+ 24.0	—	» $K = 0$
K-type	486	− 0.204	+ 3.515	− 2.128	4.115	273.3	+ 31.1	+ 0.75	»
»	»	− 0.155	+ 3.489	− 2.369	4.220	272.5	+ 34.2	—	» $K = 0$
M-type	85	− 0.004	+ 3.893	− 2.108	4.427	270.0	+ 28.4	+ 1.11	»
»	»	+ 0.165	+ 3.935	− 2.376	4.600	267.5	+ 31.1	—	» $K = 0$

TABLE IV b.

Magn	Number of stars	U_0''	V_0''	W_0''	S	Apex α	Apex δ	K	Remarks
magn $\leqq$ 4.9: stars surrounding the pole of the Milky Way	458	− 0.096	+ 3.406	− 1.822	3.864	271°.6	+ 28°.1	+ 0.46	from the means in the squares.
magn $\leqq$ 4.9: stars in the plane of the Milky Way	656	+ 0.150	+ 3.894	− 1.906	4.338	267.8	+ 26.1	+ 0.59	»
magn $\leqq$ 4.9: stars over the whole sky	1114	− 0.096	+ 3.676	− 1.818	4.103	271.5	+ 26.3	+ 0.52	»
magn $\geqq$ 5.0	482	+ 0.043	+ 3.825	− 2.263	4.444	269.4	+ 30.6	+ 0.18	individual
»	»	+ 0.047	+ 3.830	− 2.313	4.474	269.3	+ 31.1	—	» $K = 0$
all stars surrounding the pole of the M. W.	655	− 0.083	+ 3.503	− 2.069	4.069	271.4	+ 30.6	+ 0.48	from the means in the squares.
all stars in the plane of the Milky Way	941	+ 0.362	+ 3.966	− 2.133	4.518	264.8	+ 28.2	+ 0.51	»
all stars over the whole sky	1596	− 0.032	+ 3.676	− 2.005	4.187	270.5	+ 28.6	+ 0.50	»

to differ very little, at least concerning the direction of the solar motion, in spite of the uncertainty that will occur on account of the small angular distance of the apex from the plane of the Milky Way.

Concerning the relative velocity of the sun, it seems to be larger for stars in the galactic plane than for stars surrounding the pole. This fact may, indeed, largely be due to the relatively large number of class *B* stars in the plane of the Milky Way. These stars show, as seen in table IV a, the largest value of *S*.

16. A general wiew of the results for stars of different spectral classes seems to indicate exceptionally high velocities for classes *B* and *M*. The mean error is however large, as will be shown in a following paragraph.

For the different spectral classes Campbell has computed the sun's relative velocity[1]. It is, however, to be observed that the solutions are different so far as Campbell determines the solar velocities, assuming the position of the apices to be known for each type. For class *B* stars he accepts the apex direction $\alpha = 270^0.5 \quad \delta = +34^0.3$ found from Boss' general solution from all spectral types. For the other types he assumes the apex direction: $\alpha = 270^0.0 \quad \delta = +30^0.0$.

From the equation

$$\cos d\, V_0 - V = 0,$$

where V_0 is the velocity of the solar system with reference to the stars, d the angular distances of the stars from the assumed apex and further V the observed velocities of the stars or the groups into which the stars were divided, the unknown V_0 was solved with aid of the method of least squares.

The results of Campbell do not differ very much from the present. On the whole the stars used are also the same.

The constant *K*.

17. Concerning the constant K it enters the general equations for the apex elements, and is solved with the method of least squares. Campbell gets his values from a simple mean of the residual velocities, or the velocities corrected for the influence of the solar motion. To get a comparison with his values I have in table V reprinted the two series, adding the mean error to the present results.

The agreement seems to be good when taking account of the mean errors.

In fig. 1 I have made a graphical representation of this constant. As remarked by

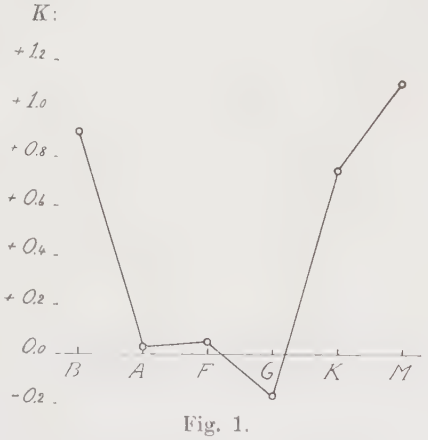

Fig. 1.
The constant K for each spectraltype.

[1] Lick. Obs. Bull. VI. **196**. p. 127.

TABLE V.

Comparison with the values of the constant K obtained by Campbell (for each spectral class).

Spectral class	K (Gyllenberg)	K (Campbell)
B	$+0.91 \pm 0.11$	$\begin{cases} +0.832 \\ +0.859 \end{cases}$
A	$+0.03 \pm 0.18$	$+0.201$
F	$+0.05 \pm 0.23$	$+0.013$
G	-0.17 ± 0.28	$\begin{cases} -0.042 \\ +0.046 \end{cases}$
K	$+0.75 \pm 0.19$	$\begin{cases} +0.403 \\ +0.595 \end{cases}$
M	$+1.11 \pm 0.45$	$\begin{cases} +0.830 \\ +0.969 \end{cases}$

Campbell it seems probable that the stars of type B—$B5$ have a larger value of K than stars of type $B8$—$B9$. It would thus be possible from a smoothed curve to interpolate the value of K for the subdivisions of the spectral classes. From a preliminary computation of the constant K within class B stars (page 24) nothing seems however to indicate a continuous variation when passing from class $B0$ to class $B9$.

18. In his paper »On the Motion of the brighter class B stars», Campbell has tested the value of this constant in zones at different angular distances from the apex. No variation was however observed. In order to test the value of K at different distances from the vertex and furthermore in the plane of the Milky Way, I have in the figures 2—8, representing the galactic plane, illustrated the value of K. The numbers denote the galactic longitudes and are computed from the vertex (closer defined in the following Chapter III) $\alpha = 274^0.6 \quad \delta = -12^0.0$. From the dotted line the values of K are marked out, positive values in the direction from, and negative values in the direction of the centre. The values were computed by subtracting the solar velocity components from the mean displacements in the squares.

The variation of K seems quite irregular with the exception of the types B and M, where a distinct relation with angular distance from the vertices, situated at 0^0 and 180^0 seems to exist. The mean velocity of the class B stars being small and consequently also the mean error of K in the squares, the numerical value of this constant seems to be in some way dependent on the position of the stars in relation to the vertex. In the same manner the stars of type M show an identical relation. It is true that the mean error here is large and the number of stars small, but the difference in the means of K from stars in a zone surrounding the vertices and in a zone at angular distance of 90^0 gives evidence of a large variation.

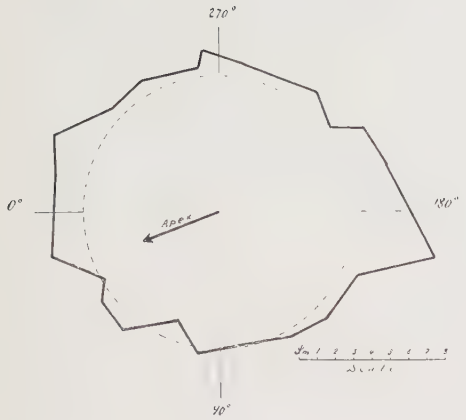

Fig. 2.
Class *B* stars.

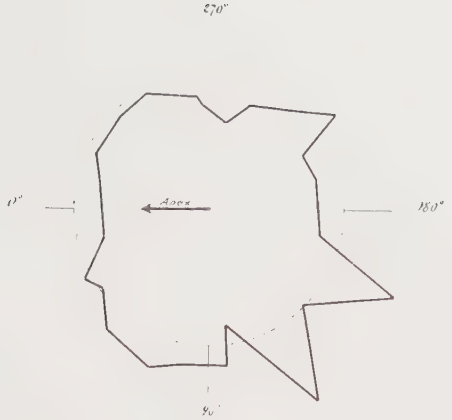

Fig. 3.
Class *A* stars.

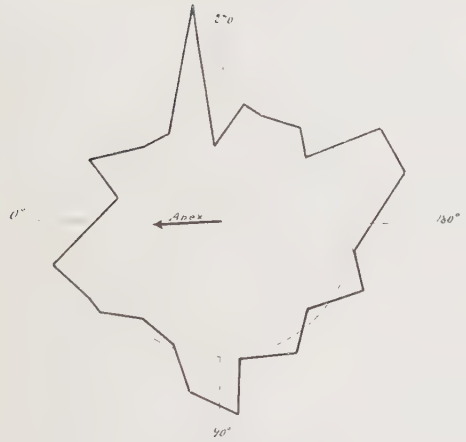

Fig. 4.
Class *F* stars.

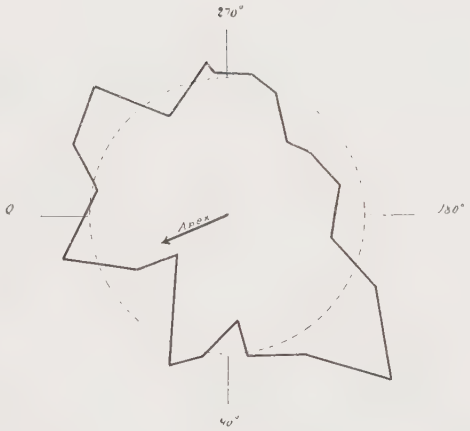

Fig. 5.
Class *G* stars.

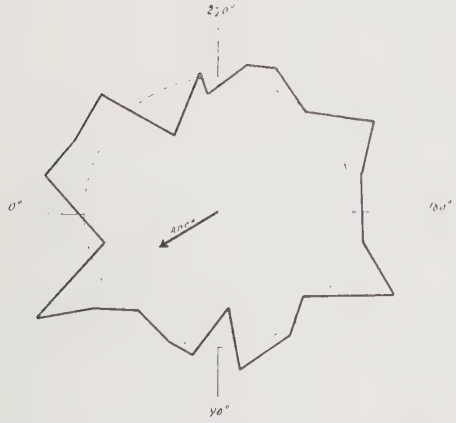

Fig. 6.
Class *K* stars.

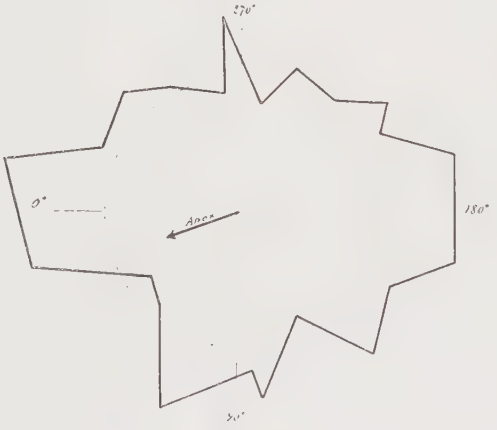

Fig. 7.
Class *M* stars.

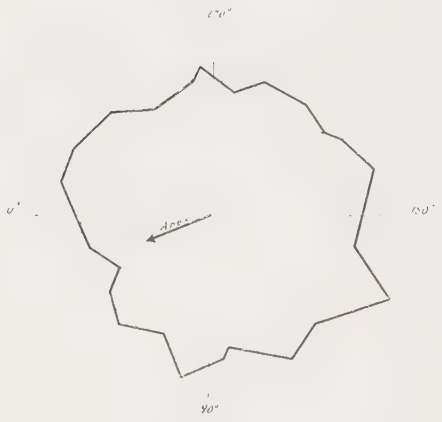

Fig. 8.
Stars of magn. ≤ 4.9.

Assuming however the stars of types $B0$ to $B2$ to have their constant K larger than the other class B stars, the variation of K in galactic longitude might be caused by a clustering of these stars about the vertices.

The stars used were however distributed in the following way:

	$B0-B1$	$B2$	$B3$	$B5$	$B8-B9$
No. of stars in the M. W.	37	24	72	42	41
No. of stars surrounding the vertices	12	6	12	10	11

The values of K for the subdivisions of the class B stars were obtained from the numerical mean value of the peculiar velocities, the relative velocity of the sun being taken from table IV a. The computation led to the following values of K:

	K (magn. ≤ 4.9)		K (all magn.)	
$B0-B1$	$+1.22$	± 0.14	$+1.00$	± 0.15
$B2$	$+0.84$	± 0.13	$+0.77$	± 0.12
$B3$	$+1.10$	± 0.15	$+0.89$	± 0.12
$B5$	$+1.18$	± 0.14	$+1.29$	± 0.12
$B8-B9$	$+0.10$	± 0.18	$+0.26$	± 0.14
all stars	$+0.92$	± 0.07	$+0.87$	± 0.06

There is a remarkable difference in the value of K for stars of type $B8—B9$ and for the other class B stars. However this circumstance and the relative distribution of the class B stars seem insufficient to explain the variation of K in terms of galactic longitudes.

19. Concerning the origin of this remarkable constant, Campbell says that a personal equation in the measurement of the spectrogram, systematically positive amounting to 5 km per second cannot be regarded as possible, while observations

of class B stars at other observatories are in good accordance with those obtained at *Mount Hamilton* and in *Chile*. The most probable explanation however seems to him to be found in the atmospheric conditions of the stars. In another paper [1] he calls attention to the slight relative displacements towards red, found by Mr. Jewell for many of the spectral lines in the solar spectrum. Analogous phenomena may exist for the class B stars, with the difference, however, that on the sun

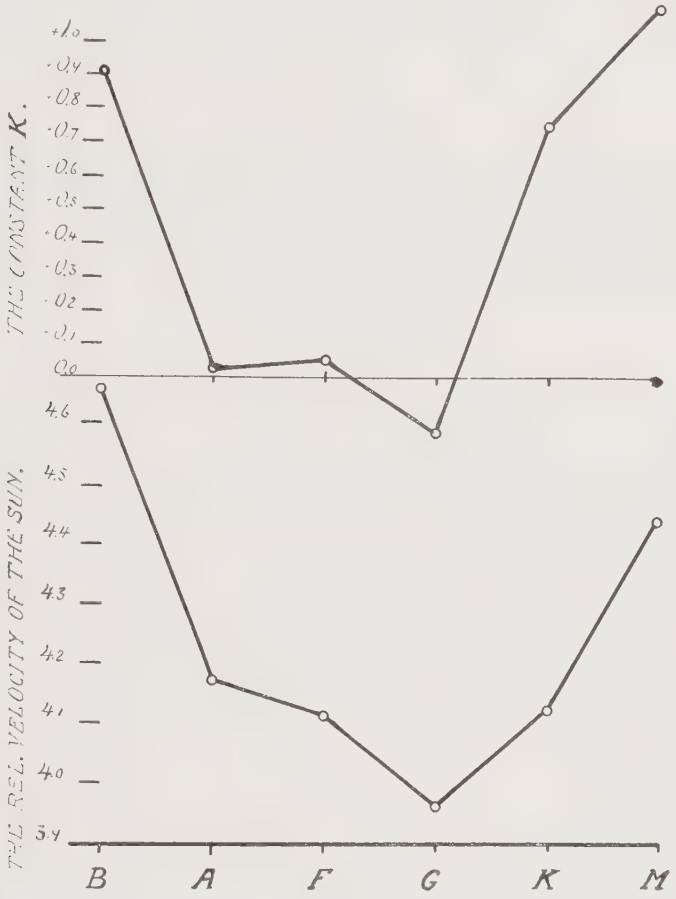

Fig. 9.

we observe the vapours of the heavier elements, whereas in class B stars we observe brighter gases, principally helium and hydrogen. In this way however the large values of K for the spectral classes K and M will not be explained.

20. The assumption of the existence of a systematic constant K seems strongly to influence the relative velocity of the sun, as well as the apex direction, at least when K has a large value. It is however remarkable, that the change of the apex direction, when K is put equal to zero, corresponds to an increase of the galactic latitude, the galactic longitude being unaltered.

[1] Note on the evidences of rapid convection in Stellar atmospheres. Lick. Obs. Bull. VIII. No. **257**, page 82.

On the other hand the value of K seems to be strongly correlated with the relative solar velocity.

In fig. 9 I have made a graphical comparison between the relative velocity of the sun and the value of K; the agreement is very striking. It is, besides, interesting, that the values of the distances of the stars of different spectral classes will, drawn as a curve, have the same form, when the parallaxes are computed from a comparison between proper motion and radial velocity. The mean parallaxes are indeed influenced by the assumed value of the solar velocity. This will however not change the mutual relations.

The mean errors in the solar velocity determinations.

21. In the linear equations in U_0'', V_0'', W_0'' and K, the mean error in one unknown quantity — R — is expressed through

$$\varepsilon(R) = \frac{\varepsilon}{\sqrt{p_R}}$$

where p_R denotes the weight of R obtained in the usual manner. Further ε is the mean error in one single observation and was put equal to σ (see table X page 30). The stars in the squares taken in groups, the mean error of a single observation is equal to the mean error in the mean or

$$\varepsilon = \sigma : \sqrt{n}\,.$$

The number n was put equal to the mean number of stars in the squares.

The following table gives the mean errors computed in this way. In order to determine the mean error in the apex direction it was assumed that

$$\varepsilon(U_0'') = \varepsilon(V_0'') = \varepsilon(W_0'') = \varepsilon(S).$$

TABLE VI.

The mean errors in the coordinates of the apex, the solar velocity and the constant K.

Type	$\varepsilon(U_0'')$	$\varepsilon(V_0'')$	$\varepsilon(W_0'')$	$\varepsilon(S)$	$\varepsilon(D)$	$\varepsilon(D)$ in degrees	$\varepsilon(A)$	$\varepsilon(A)$ in degrees
B	0.218	0.170	0.178	0.190	0.041	2°.4	0.052	2°.9
A	0.304	0.290	0.321	0.305	0.060	3°.4	0.084	4°.8
F	0.450	0.401	0.412	0.421	0.100	5°.7	0.116	6°.6
G	0.498	0.455	0.479	0.474	0.113	6°.5	0.130	7°.4
K	0.346	0.325	0.313	0.328	0.078	4°.5	0.090	5°.2
M	0.772	0.831	0.763	0.789	0.188	10°.8	0.216	12°.4
m $\leq$ 4.9	—	—	—	0.182	0.043	2°.5	0.050	2°.9
m $\geq$ 5.0	0.348	0.324	0.294	0.322	0.077	4°.4	0.089	5°.1
all stars	—	—	—	0.160	0.038	2°.2	0.044	2°.5

CHAPTER III

The mean velocities of stars from observations in the line of sight.

22. Before any further computations were made, the stars mentioned in the first chapter, whose velocities were marked as particularly uncertain, were rejected. By determining the apex and the solar velocity small errors like these in the catalogue do not influence the solution in so high a degree as by the treatment of the moments of the second order.

The distribution of the remaining stars is shown in the table VII.

TABLE VII.

Distribution of the stars used by determining the velocity distribution and the mean velocities.

Square	n	Square	n
A_1	17	D_1	17
A_2	29	D_2	23
		D_3	38
B_1	39	D_4	36
B_2	34	D_5	24
B_3	25	D_6	12
B_4	27	D_7	15
B_5	30	D_8	23
B_6	24	D_9	36
B_7	24	D_{10}	31
B_8	34	D_{11}	29
B_9	41	D_{12}	19
B_{10}	33		
		E_1	35
C_1	23	E_2	38
C_2	28	E_3	44
C_3	51	E_4	63
C_4	32	E_5	53
C_5	23	E_6	50
C_6	20	E_7	48
C_7	30	E_8	44
C_8	31	E_9	36
C_9	22	E_{10}	35
C_{10}	39		
C_{11}	29	F_1	38
C_{12}	30	F_2	29

23. Furthermore, the velocities of stars of spectral classes B, K and M were corrected on account of the constant K, this being assumed to have the nature of a systematic error in the measurements.

The following corrections were applied [1]:

Type	B	K	M
Correction	— 0.91	— 0.75	— 1.11

For the other spectral classes no corrections were applied, because in these cases the mean error in K was too large.

24. The stars thus being freed from systematic errors, there is nothing to prevent a diametrical combination of the squares. Indeed, all calculations in this chapter are performed so that the stars of south declination are transferred to the diametrically situated point on the north sky, the signs of the velocities at the same time being changed.

[1] The values of K on page 24 seem to be constant for all class B stars except for the stars of type $B8$—$B9$. This table, however, was computed after the writing of the present chapter, for which reason the correction — 0.91 was applied to all class B stars.

TABLE VIII.

Direction cosines of the apex ($\alpha = 270^0$ $\delta = +30^0$) in relation to the system K_3.

Square	c_{13}	c_{23}	c_{33}
A_1	0.	+ 0.9396	+ 0.3121
A_2	0.	— 0.7660	+ 0.6427
B_1	— 0.8237	+ 0.5426	+ 0.1653
B_2	— 0.5090	+ 0.8492	— 0.1404
B_3	0.	+ 0.9664	— 0.2571
B_4	+ 0.5090	+ 0.8492	— 0.1404
B_5	+ 0.8237	+ 0.5426	+ 0.1653
B_6	+ 0.8237	+ 0.1634	+ 0.5431
B_7	+ 0.5090	— 0.1432	+ 0.8488
B_8	0.	— 0.2604	+ 0.9655
B_9	— 0.5090	— 0.1432	+ 0.8488
B_{10}	— 0.8237	+ 0.1634	+ 0.5431
C_1	— 0.8365	+ 0.5402	— 0.0920
C_2	— 0.6123	+ 0.6373	— 0.4680
C_3	— 0.2241	+ 0.6933	— 0.6850
C_4	+ 0.2241	+ 0.6933	— 0.6850
C_5	+ 0.6123	+ 0.6373	— 0.4680
C_6	+ 0.8365	+ 0.5402	— 0.0920
C_7	+ 0.8365	+ 0.4282	+ 0.3420
C_8	+ 0.6123	+ 0.3311	+ 0.7180
C_9	+ 0.2241	+ 0.2751	+ 0.9350
C_{10}	— 0.2241	+ 0.2751	+ 0.9350
C_{11}	— 0.6123	+ 0.3311	+ 0.7180
C_{12}	— 0.8365	+ 0.4282	+ 0.3420
D_1	— 0.8365	+ 0.4282	— 0.3420
D_2	— 0.6123	+ 0.3311	— 0.7180
D_3	— 0.2241	+ 0.2751	— 0.9350
D_4	+ 0.2241	+ 0.2751	— 0.9350
D_5	+ 0.6123	+ 0.3311	— 0.7180
D_6	+ 0.8365	+ 0.4282	— 0.3420
D_7	+ 0.8365	+ 0.5402	+ 0.0920
D_8	+ 0.6123	+ 0.6373	+ 0.4680
D_9	+ 0.2241	+ 0.6933	+ 0.6850
D_{10}	— 0.2241	+ 0.6933	+ 0.6850
D_{11}	— 0.6123	+ 0.6373	+ 0.4680
D_{12}	— 0.8365	+ 0.5402	+ 0.0920
E_1	— 0.8237	+ 0.1634	— 0.5431
E_2	— 0.5090	— 0.1432	— 0.8488
E_3	0.	— 0.2604	— 0.9655
E_4	+ 0.5090	— 0.1432	— 0.8488
E_5	+ 0.8237	+ 0.1634	— 0.5431
E_6	+ 0.8237	+ 0.5426	— 0.1653
E_7	+ 0.5090	+ 0.8492	+ 0.1404
E_8	0.	+ 0.9664	+ 0.2571
E_9	— 0.5090	+ 0.8492	+ 0.1404
E_{10}	— 0.8237	+ 0.5426	— 0.1653
F_1	0.	— 0.7660	— 0.6427
F_2	0.	+ 0.9396	— 0.3421

Now for each square and for each spectral class and magnitude the values of Σz^2 were calculated. In the usual way we may now compute the mean deviation. The stars in the different squares however being small, I have not for this purpose used the observed mean, but computed the deviation about a theoretical mean ζ_0 — that is the value of z obtained for every square and spectral class as the component of the solar motion and defined in the following way:

$$\zeta_0 = S \cos. \text{ the angle (apex — square).}$$

Here the sun's relative velocity, S, is to be taken from the table IV. As for the position of the apex, this was assumed to be $\alpha = 270^0$, $\delta = +30^0$, the same for all types, because a variation in the position of the apex will in a much smaller degree influence the value ζ_0 than a variation in the relative velocity of the sun.

The cosines of the angles between the apex and the centres of the squares may be taken from the table VIII, where the direction cosines of this assumed apex in relation to the system K_3 are tabulated.

The following designations are used:

$$c_{13} = \cos (\text{apex} - X),$$
$$c_{23} = \cos (\text{apex} - Y),$$
$$c_{33} = \cos (\text{apex} - Z).$$

25. The quantities, in the following denoted as dispersions, are all calculated from the formula

$$n\sigma^2 = \Sigma z^2 - n\zeta_0^2 - 2\zeta_0 \Sigma z, \tag{18}$$

which formula for $\zeta_0 = z_0 = \Sigma z : n$ is transformed into the general expression for the dispersion about the mean.

The table IX below contains the values of σ obtained in the manner described.

A short glance at the table shows that the mean velocity within the different spectral groups is very varying without any conspicuous regularity.

TABLE IX.

The mean velocities of the different spectral classes, computed after the combination of the diametrically situated squares.

Square	*B*		*A*		*F*		*G*		*K*		*M*	
	n	σ	*n*	σ	*n*	σ	*n*	σ	*n*	σ	*n*	σ
$A_1 + F_2$	2	+ 0.90	6	+ 3.38	7	+ 3.26	4	+ 3.27	22	+ 4.06	5	+ 2.60
$A_2 + F_1$	7	+ 1.62	5	+ 2.94	10	+ 4.14	11	+ 3.74	31	+ 4.56	3	+ 5.14
$B_1 + E_6$	32	+ 1.97	12	+ 2.79	10	+ 5.30	9	+ 3.09	21	+ 4.38	5	+ 3.74
$B_2 + E_7$	29	+ 2.14	3	+ 2.38	13	+ 2.90	13	+ 4.41	19	+ 4.32	4	+ 4.47
$B_3 + E_8$	12	+ 2.02	12	+ 3.19	9	+ 4.22	8	+ 5.87	24	+ 4.69	4	+ 3.35
$B_4 + E_9$	6	+ 0.96	15	+ 2.35	11	+ 3.79	9	+ 3.62	20	+ 3.91	2	+ 5.55
$B_5 + E_{10}$	2	+ 1.71	16	+ 3.17	10	+ 3.57	9	+ 4.55	23	+ 4.36	5	+ 2.40
$B_6 + E_1$	4	+ 2.28	17	+ 2.04	12	+ 2.41	8	+ 3.59	13	+ 3.95	5	+ 3.65
$B_7 + E_2$	6	+ 1.92	10	+ 3.07	13	+ 3.61	10	+ 3.77	19	+ 3.67	4	+ 4.70
$B_8 + E_3$	13	+ 1.35	12	+ 1.99	13	+ 2.79	8	+ 4.20	26	+ 3.88	6	+ 2.88
$B_9 + E_4$	28	+ 1.63	12	+ 2.11	16	+ 2.50	10	+ 2.46	36	+ 3.68	2	+ 0.85
$B_{10} + E_5$	20	+ 2.28	14	+ 3.33	11	+ 3.98	10	+ 2.35	27	+ 3.53	3	+ 10.93
$C_1 + D_7$	3	+ 1.13	8	+ 1.51	4	+ 2.28	7	+ 3.08	13	+ 4.30	3	+ 5.26
$C_2 + D_8$	11	+ 1.32	9	+ 4.24	5	+ 3.69	4	+ 4.04	20	+ 4.39	2	+ 6.00
$C_3 + D_9$	17	+ 1.38	23	+ 4.49	18	+ 4.65	9	+ 3.46	16	+ 2.88	4	+ 1.36
$C_4 + D_{10}$	6	+ 2.32	8	+ 3.96	8	+ 3.57	15	+ 3.60	22	+ 5.59	3	+ 5.08
$C_5 + D_{11}$	2	+ 2.07	13	+ 3.63	9	+ 4.54	11	+ 4.79	13	+ 2.28	4	+ 4.77
$C_6 + D_{12}$	--	—	11	+ 2.89	8	+ 3.88	5	+ 1.84	11	+ 4.62	4	+ 5.13
$C_7 + D_1$	1	+ 0.17	8	+ 3.91	5	+ 3.37	8	+ 3,57	20	+ 3.99	5	+ 4.28
$C_8 + D_2$	4	+ 1.95	10	+ 2.42	11	+ 4.54	6	+ 7.69	20	+ 4.81	3	+ 6.66
$C_9 + D_3$	16	+ 1.34	9	+ 3.59	8	+ 4.50	6	+ 7.89	18	+ 3.73	2	+ 9.13
$C_{10} + D_4$	18	+ 2.43	12	+ 3.03	9	+ 5.03	11	+ 3.74	22	+ 4.46	2	+ 3.50
$C_{11} + D_5$	4	+ 1.27	9	+ 2.08	11	+ 4.05	13	+ 4.46	14	+ 4.63	2	+ 1.17
$C_{12} + D_6$	4	+ 1.80	9	+ 2.36	6	+ 2.15	4	+ 3.42	16	+ 4.52	3	+ 3.12

26. Let us start with an examination of the mean velocities of the stars in relation to their spectral classes. For this purpose all squares have to be combined. The dispersion being computed about a fixed point in every square, the formula takes the simple form

$$N\sigma_s^2 = n_1\sigma_1^2 + n_2\sigma_2^2 + \ldots + n_{24}\sigma_{24}^2.$$

Here N denotes the total number of stars, n_i and σ_i are referred to the individual squares.

The table X contains these *mean velocities* σ_s for the spectral classes and for two groups of magnitudes.

For every value the mean error is added, computed from the usual formula

$$\varepsilon(\sigma) = \frac{\sigma}{\sqrt{2N}}. \tag{10}$$

The column designated with ϑ contains the *average velocity*, and is related with the former through

$$\vartheta = \sqrt{\frac{2}{\pi}}\,\sigma. \tag{11}$$

TABLE X.

The mean velocity and its variation with spectral classes.

Type	Number of stars	Mean velocity	ϑ	Ω	Results of Campbell	
					Number of stars	Average velocity
B	247	1.854 ± 0.083	1.479 ± 0.066	2.958 ± 0.132	225	1.366 — 1.410
A	263	3.102 ± 0.149	2.475 ± 0.119	4.950 ± 0.238	177	2.215 — 2.312
F	237	3.816 ± 0.175	3.045 ± 0.140	6.090 ± 0.280	185	3.034
G	208	4.173 ± 0.204	3.330 ± 0.163	6.660 ± 0.326	123—128	2.723 — 3.160
K	85	4.547 ± 0.348	3.628 ± 0.278	7.256 ± 0.556	70—73	3.251 — 3.618
M	486	4.200 ± 0.135	3.351 ± 0.108	6.702 ± 0.216	369—382	3.188 — 3.547
magn $\leq$ 4.9	1069	3.527 ± 0.076	2.814 ± 0.061	5.624 ± 0.122	—	—
magn $\geq$ 5.0	462	4.094 ± 0.135	3.267 ± 0.108	6.534 ± 0.216	—	—
all stars	1531	3.623 ± 0.066	2.891 ± 0.053	5.782 ± 0.106	—	—

The average velocities are the values determined by CAMPBELL and in the last column I have for a comparison reprinted his results and expressed them in the units used here. The remaining column gives the value of Ω that is the *mean absolute velocity* defined through

$$\Omega = 2\vartheta = 2\sqrt{\frac{2}{\pi}}\sigma. \tag{12}$$

Concerning the mean velocities of brighter and fainter stars the latter give a much higher value than the former, this fact may however exclusively be due to the large number of stars of so called later types that enter the group magn $\geq$ 5.0.

The ellipsoidal hypothesis in its generalized form.

27. Returning to the table IX, these values may be used for a closer examination of the velocity distribution. To this end the ellipsoidal hypothesis was adopted in its generalized form. In another paper I have given a short description of the solution and preliminary derived the numerical results [1].

As to the developement of the formulas and the expressions I have used the same signification as CHARLIER in his memoir. It was however necessary when studying the three axial distribution of the velocities to introduce three variables.

I give here only the expressions for the moments about the origin and about the mean.

The former is denoted by:

$$N'_{ijk} = \iiint_{-\infty}^{+\infty} dU\, dV\, dW\, \varphi(U, V, W)\, U^i V^j W^k, \tag{13}$$

[1] Meddelande fr. Lunds Astr. Obs. N:o **59**.

and the latter

$$N_{ijk} = \iiint_{-\infty}^{+\infty} dU\, dV\, dW\, \varphi(U, V, W)\,(U-U_0)^i\,(V-V_0)^j\,(W-W_0)^k. \tag{14}$$

Hereby the significations below are explained.

28. Assume the velocities of the stars to be distributed according to the generalized law of MAXWELL and let U', V', W' denote the three velocity components of a star relative to the sun, then the number of stars with components of velocity between the limits

$$U' \pm {}^1/_2\, dU', \quad V' \pm {}^1/_2\, dV', \quad W' \pm {}^1/_2\, dW'$$

is given through the formula

$$e^{-{}^1/_2 (A'U'^2 + B'V'^2 + C'W'^2)}\, dU'\, dV'\, dW'. \tag{15}$$

The surface

$$A'U'^2 + B'V'^2 + C'W'^2 = \text{const}$$

is called the velocity ellipsoid and the coefficients have the following expression

$$A' = \frac{1}{\sigma_1'^2}, \quad B' = \frac{1}{\sigma_2'^2}, \quad C' = \frac{1}{\sigma_3'^2},$$

where σ_1', σ_2' and σ_3' denote the dispersions in the directions of the three axes of the ellipsoid.

From the schemes page 13 we get the relation between the velocity components in the different systems. These schemes give us now:

$$\begin{aligned} U'' &= \gamma_{11} U + \gamma_{12} V + \gamma_{13} W, \\ V'' &= \gamma_{21} U + \gamma_{22} V + \gamma_{23} W, \\ W'' &= \gamma_{31} U + \gamma_{32} V + \gamma_{33} W, \end{aligned} \tag{16}$$

and

$$\begin{aligned} U &= \gamma_{11} U'' + \gamma_{21} V'' + \gamma_{31} W'', \\ V &= \gamma_{12} U'' + \gamma_{22} V'' + \gamma_{32} W'', \\ W &= \gamma_{13} U'' + \gamma_{23} V'' + \gamma_{33} W'', \end{aligned} \tag{16*}$$

likewise

$$\begin{aligned} U'' &= \varepsilon_{11} U' + \varepsilon_{12} V' + \varepsilon_{13} W', \\ V'' &= \varepsilon_{21} U' + \varepsilon_{22} V' + \varepsilon_{23} W', \\ W'' &= \varepsilon_{31} U' + \varepsilon_{32} V' + \varepsilon_{33} W', \end{aligned} \tag{17}$$

and

$$\begin{aligned} U' &= \varepsilon_{11} U'' + \varepsilon_{21} V'' + \varepsilon_{31} W'', \\ V' &= \varepsilon_{12} U'' + \varepsilon_{22} V'' + \varepsilon_{32} W'', \\ W' &= \varepsilon_{13} U'' + \varepsilon_{23} V'' + \varepsilon_{33} W''. \end{aligned} \tag{17*}$$

Substituting in equation (15) U', V', W' from the formulæ (17*) the exponential expression gets the form:

$$e^{-\frac{1}{2}f}, \tag{18}$$

where

$$f = A''U''^2 + B''V''^2 + C''W''^2 + 2\,D''V''W'' + 2\,E''W''U'' + 2\,F''U''V''. \tag{19}$$

Here is

$$\begin{aligned}
A'' &= A'\,\varepsilon_{11}^2 + B'\,\varepsilon_{12}^2 + C'\,\varepsilon_{13}^2,\\
B'' &= A'\,\varepsilon_{21}^2 + B'\,\varepsilon_{22}^2 + C'\,\varepsilon_{23}^2,\\
C'' &= A'\,\varepsilon_{31}^2 + B'\,\varepsilon_{32}^2 + C'\,\varepsilon_{33}^2,\\
D'' &= A'\,\varepsilon_{21}\,\varepsilon_{31} + B'\,\varepsilon_{22}\,\varepsilon_{32} + C'\,\varepsilon_{23}\,\varepsilon_{33},\\
E'' &= A'\,\varepsilon_{31}\,\varepsilon_{11} + B'\,\varepsilon_{32}\,\varepsilon_{12} + C'\,\varepsilon_{33}\,\varepsilon_{13},\\
F'' &= A'\,\varepsilon_{11}\,\varepsilon_{21} + B'\,\varepsilon_{12}\,\varepsilon_{22} + C'\,\varepsilon_{13}\,\varepsilon_{23}.
\end{aligned} \tag{20}$$

Our problem is now to determine the coefficients A'', B'', C'', D'', E'', F'' from the observations.

Turning the system of coordinates so that it coincides with the K_3-system or in other words substituting the values (16) in (19) we get:

$$f = AU^2 + BV^2 + CW^2 + 2\,DVW + 2\,EWU + 2\,FUV, \tag{21}$$

where

$$\begin{aligned}
A &= A''\gamma_{11}^2 + B''\gamma_{21}^2 + C''\gamma_{31}^2 + 2\,D''\gamma_{21}\gamma_{31} + 2\,E''\gamma_{31}\gamma_{11} +\\
&\quad + 2\,F''\gamma_{11}\gamma_{21},\\
B &= A''\gamma_{12}^2 + B''\gamma_{22}^2 + C''\gamma_{32}^2 + 2\,D''\gamma_{22}\gamma_{32} + 2\,E''\gamma_{32}\gamma_{12} +\\
&\quad + 2\,F''\gamma_{12}\gamma_{22},\\
C &= A''\gamma_{13}^2 + B''\gamma_{23}^2 + C''\gamma_{33}^2 + 2\,D''\gamma_{23}\gamma_{33} + 2\,E''\gamma_{33}\gamma_{13} +\\
&\quad + 2\,F''\gamma_{13}\gamma_{23},\\
D &= A''\gamma_{12}\gamma_{13} + B''\gamma_{22}\gamma_{23} + C''\gamma_{32}\gamma_{33} + D''(\gamma_{22}\gamma_{33} + \gamma_{32}\gamma_{23}) +\\
&\quad + E''(\gamma_{32}\gamma_{13} + \gamma_{33}\gamma_{12}) + F''(\gamma_{12}\gamma_{23} + \gamma_{13}\gamma_{22}),\\
E &= A''\gamma_{13}\gamma_{11} + B''\gamma_{23}\gamma_{21} + C''\gamma_{33}\gamma_{31} + D''(\gamma_{23}\gamma_{31} + \gamma_{33}\gamma_{21}) +\\
&\quad + E''(\gamma_{33}\gamma_{11} + \gamma_{31}\gamma_{13}) + F''(\gamma_{13}\gamma_{21} + \gamma_{11}\gamma_{23}),\\
F &= A''\gamma_{11}\gamma_{12} + B''\gamma_{21}\gamma_{22} + C''\gamma_{31}\gamma_{32} + D''(\gamma_{21}\gamma_{32} + \gamma_{31}\gamma_{22}) +\\
&\quad + E''(\gamma_{31}\gamma_{12} + \gamma_{32}\gamma_{11}) + F''(\gamma_{11}\gamma_{22} + \gamma_{12}\gamma_{21}).
\end{aligned} \tag{22}$$

29. In order to get a solution of the problem exclusively from observations in the line of sight, i. e. the Z-axes, we have to integrate the function (18) over all values for U and V, f having the form (21).

Performing this integration we get

$$\Phi_0 = K \cdot e^{-\frac{1}{2}HW^2}, \tag{23}$$

where

$$H = \frac{ABC - AD^2 - BE^2 - CF^2 + 2DEF}{AB - F^2}. \tag{24}$$

Substituting the relation (22) in (24) we get

$$H = \frac{A''B''C'' - A''D''^2 - B''E''^2 - C''F''^2 + 2D''E''F''}{AB - F^2}.$$

Here

$$\begin{aligned} AB - F^2 = {} & (B''C'' - D''^2)\,\gamma_{13}^{\,2} + (C''A'' - E''^2)\,\gamma_{23}^{\,2} \\ & + (A''B'' - F''^2)\,\gamma_{33}^{\,2} + 2\,(D''E'' - C''F'')\,\gamma_{13}\,\gamma_{23} \\ & + 2\,(E''F'' - A''D'')\,\gamma_{23}\,\gamma_{33} + 2\,(F''D'' - B''E'')\,\gamma_{33}\,\gamma_{13}. \end{aligned}$$

In the expression for H, the numerator does not contain the direction cosines of the squares and is therefore a constant.

Observing that

$$H = \frac{1}{\sigma^2}, \tag{27}$$

and

$$\sigma^2 = N_{002}, \tag{27*}$$

where σ denotes the dispersion in the frequency distribution of the observed radial velocities, and putting

$$\begin{aligned} \Delta &= A''B''C'' - A''D''^2 - B''E''^2 - C''F''^2 + 2\,D''E''F'', \\ &= \begin{vmatrix} A'', & F'', & E'' \\ F'', & B'', & D'' \\ E'', & D'', & C'' \end{vmatrix} = \text{const.} \end{aligned} \tag{28}$$

the equation to be solved takes the form:

$$\begin{aligned} \Delta\, N_{002} = {} & (B''C'' - D''^2)\,\gamma_{13}^{\,2} + (C''A'' - E''^2)\,\gamma_{23}^{\,2} \\ & + (A''B'' - F''^2)\,\gamma_{33}^{\,2} + 2\,(D''E'' - C''F'')\,\gamma_{13}\,\gamma_{23} \\ & + 2\,(E''F'' - A''D'')\,\gamma_{23}\,\gamma_{33} + 2\,(F''D'' - B''E'')\,\gamma_{33}\,\gamma_{13}. \end{aligned} \tag{29}$$

30. Integrating however the expression (18) over the other axes one by one, the moments of the second order expressed in the constants of the ellipsoid are easily found. The following six relations are thus obtained:

$$\begin{aligned} \Delta\, N_{200} &= BC - D^2, \\ \Delta\, N_{020} &= CA - E^2, \\ \Delta\, N_{002} &= AB - F^2, \\ \Delta\, N_{011} &= EF - AD, \\ \Delta\, N_{101} &= FD - BE, \\ \Delta\, N_{110} &= DE - CF. \end{aligned} \tag{30}$$

From these relations it is possible to express the coefficients of the ellipsoid in the moments N_{ijk}. We get for these coefficients:

$$
\begin{aligned}
\Delta_1 A &= N_{020}\, N_{002} - N_{011}^2, \\
\Delta_1 B &= N_{002}\, N_{200} - N_{101}^2, \\
\Delta_1 C &= N_{200}\, N_{020} - N_{110}^2, \\
\Delta_1 D &= N_{110}\, N_{101} - N_{011}\, N_{200}, \\
\Delta_1 E &= N_{011}\, N_{110} - N_{101}\, N_{020}, \\
\Delta_1 F &= N_{101}\, N_{011} - N_{110}\, N_{002}.
\end{aligned} \tag{31}
$$

where

$$
\Delta_1 = \begin{vmatrix} N_{200}, & N_{110}, & N_{101} \\ N_{110}, & N_{020}, & N_{011} \\ N_{101}, & N_{011}, & N_{002} \end{vmatrix}. \tag{32}
$$

The relations (30) and (31) exist in fact between the coefficients and the moments when referred to any system of coordinates. It is further easily shown that the determinants in the formulæ (30) and (31) are connected through the relation

$$\Delta = 1 : \Delta_1.$$

31. Using exclusively observations in the line of sight, the third relation in (30), coincident with (29), gives us the equation of condition. Solving the unknown quantities in the right member of (29) by the method of least squares we get in the usual way the squares of the three axes of the velocity ellipsoid expressed through

$$-\frac{\Delta}{s_1}, \qquad -\frac{\Delta}{s_2}, \qquad -\frac{\Delta}{s_3}, \tag{33}$$

where s_1, s_2, s_3 are the roots of the equation

(34)

$$s^3 - (A'' + B'' + C'')\, s^2 + (A''B'' + B''C'' + C''A'' - D''^2 - E''^2 - F''^2)\, s - \Delta = 0.$$

The direction cosines λ, μ and ν of the three axes are finally given through

$$
\begin{aligned}
(A'' - s_i)\,\lambda + F''\mu + E''\nu &= 0, \\
F''\lambda + (B'' - s_i)\,\mu + D''\nu &= 0, \\
E''\lambda + D''\mu + (C'' - s_i)\,\nu &= 0,
\end{aligned} \tag{35}
$$

where for a certain axis the corresponding value of s_i is to be substituted.

32. There are twenty-four equations of condition of the form (29) to be solved. The number of stars in the different spectral classes being rather small for separate solutions, I have found it convenient to treat them in the three following groups:

Group	Spectral classes	Number of stars
1	B and A	510
2	F and G	445
3	K and M	571

As the stars in the different squares are varying in number, the weight of each equation was put equal to the number of stars entered. Further by this combination of the spectral classes, in the left member of (29) the quantity N_{002} was exchanged by:

$$n_1\sigma_1^2 + n_2\sigma_2^2$$

where n_1, σ_1 and n_2, σ_2 are referred to the two combined spectral classes. The procedure mentioned signifies indeed a neglecting of the difference of the two means, but as this is small for neighbouring spectral classes I have for practical reasons used the above expression. For the two groups »magn ≤ 4.9» and »all stars» the dispersions were computed about the observed means and the weights were considered equal for all squares.

TABLE XI.

The axes of the velocity ellipsoids and their directions from observations in the line of sight.

Type	Number of stars	Principal vertex			Secondary vertex			Third direction		
		α	δ	σ	α	δ	σ	α	δ	σ
B and *A*	510	280°.1	— 19°.5	3.102	12°.9	— 8°.0	2.728	304°.2	+ 68°.8	1.727
F and *G*	445	253°.7	+ 0°.5	4.952	343°.3	— 37°.9	3.670	344°.1	+ 53°.0	3.023
K and *M*	571	248°.9	— 14°.1	4.508	338°.5	— 2°.2	4.266	257°.2	+ 76°.0	4.003
Magn.										
magn ≤ 4.9	1069 [2]	262°.3	— 7°.2	4.201	358°.4	— 40°.5	3.473	344°.1	+ 48°.6	2.669
magn ≥ 5.0	462	259°.9	— 0°.2	4.507	349°.8	— 6°.5	4.191	348°.2	+ 83°.8	3.635
all stars	1526 [2]	264°.1	— 5°.2	4.246	357°.1	— 29°.9	3.766	336°.5	+ 59°.5	3.056

33. In table XI the results are tabulated. The column indicated by *principal vertex* gives the magnitude and the direction of the largest axis and coincides approximatively with the vertex found by SCHWARTZSCHILD [1] from the proper motions of stars ($\alpha = 273^0$, $\delta = -6^0$). The column *secondary vertex* contains the next axis, which is directed against the pole of the Milky Way. The last column finally gives the direction of the shortest axis and its length. It is remarkable that the shortest axis as well as the longest coincides approximatively with the plane of the Milky Way.

34. To get a wiew of the positions of the velocity ellipsoids I have tried to make a graphical demonstration. The figures 10—13 represent the sky and its division into squares. The dotted line and the black star indicate the position of the Milky Way (pole: $\alpha = 191^0.2$ $\delta = +18^0.0$). The ellipsoids are assumed to be drawn in the system of coordinates determined by the position of the axes, and

[1] K. SCHWARTZSCHILD: Ueber die Eigenbewegungen der Fixsterne.

[2] Including 5 stars of spectral type *Oe*.

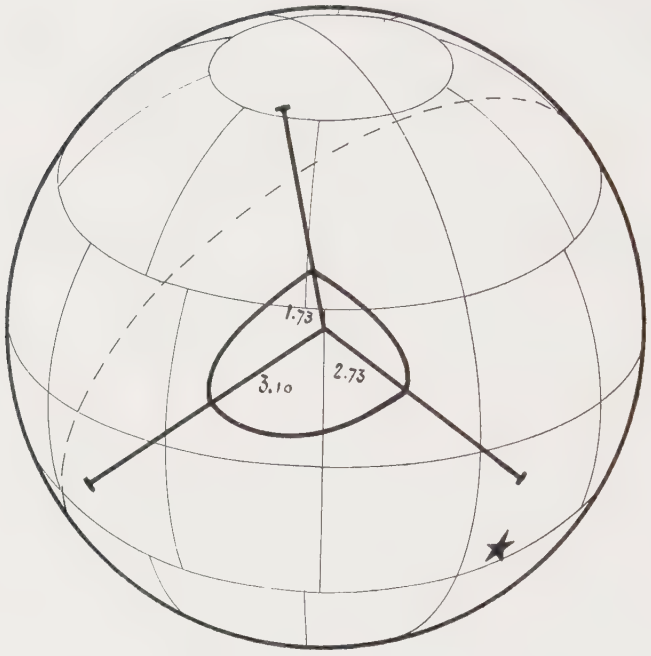

Fig. 10.

Velocity-ellipsoid from radial velocity observations of 510 stars of spectral types *B* and *A*. The black star and the dotted line indicate the position of the Milky Way, and the numbers the lengths of the axes.

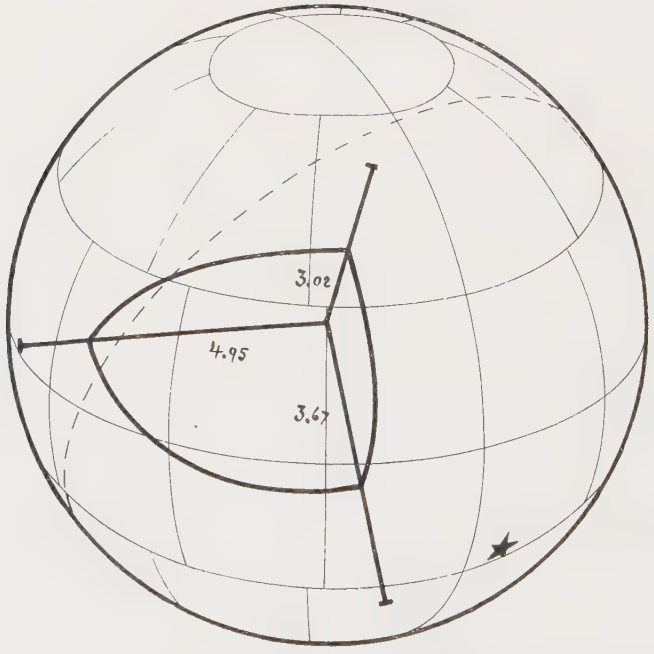

Fig. 11.

Velocity-ellipsoid from radial velocity observations of 445 stars of spectral types *F* and *G*.

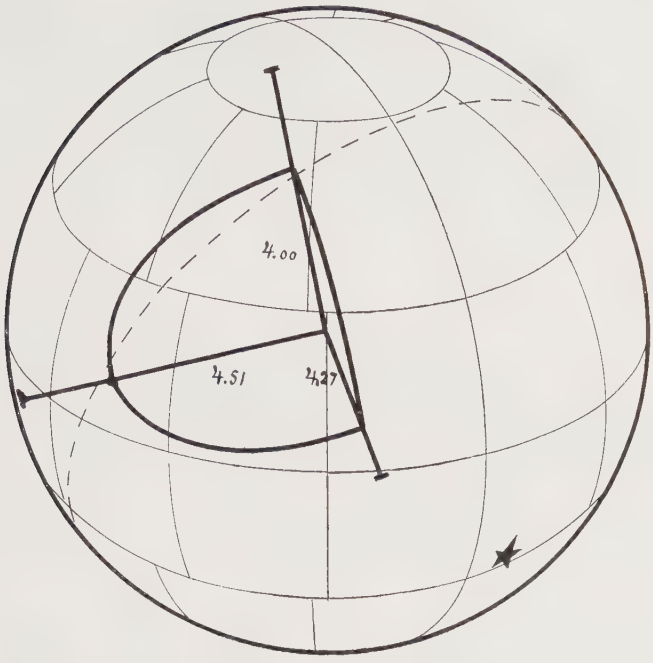

Fig. 12.

Velocity-ellipsoid from radial velocity observations of 571 stars of spectral types *K* and *M*.

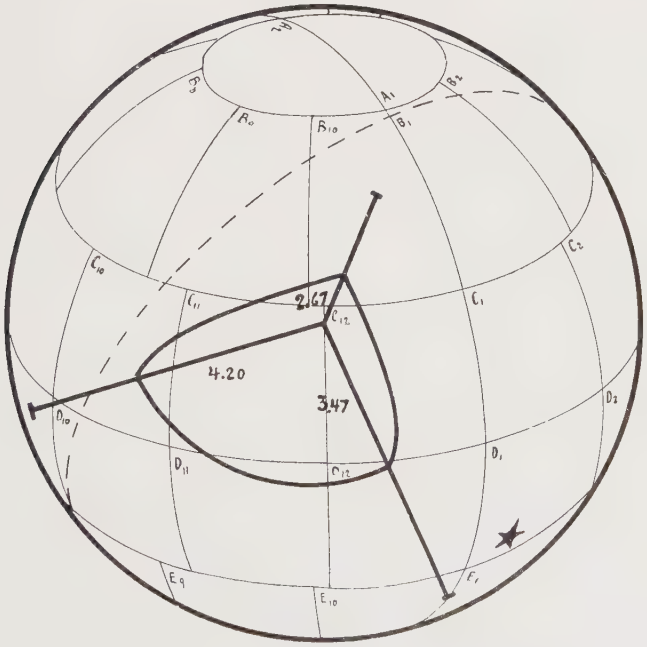

Fig. 13.

Velocity-ellipsoid from radial velocity observations of 1069 stars of magnitudes $\leq$ 4.9.

with its origin coincident with the centre of the sphere. The numbers indicate the lengths of the axes. The figure 13 contains besides the denominations of the squares.

35. In order to get an idea of to what degree the ellipsoidal hypothesis satisfies the observed data I have recalculated from the elements of the ellipsoid the dispersion in the velocity distribution for each square, and compared it with the observed value. These recalculations were easily performed with the aid of the formula (29) page 33.

TABLE XII.

Comparison between observed and recomputed dispersion in the line of sight.

Square	Number of stars	Magn $\leq$ 4.9. Dispersion		Number of stars	All stars. Dispersion	
		computed	observed		computed	observed
$A_1 + F_2$	26	3.175	3.328	46	3.337	3.513
$A_2 + F_1$	39	3.024	4.099	67	3.244	3.479
$B_1 + E_6$	67	2.961	2.763	89	3.260	3.426
$B_2 + E_7$	57	3.535	2.791	82	3.642	3.408
$B_3 + E_8$	51	3.833	3.955	69	3.888	4.179
$B_4 + E_9$	36	3.730	3.633	63	3.869	3.383
$B_5 + E_{10}$	41	3.507	3.564	65	3.747	3.821
$B_6 + E_1$	33	3.496	2.227	59	3.729	3.027
$B_7 + E_2$	42	3.597	3.419	62	3.769	3.501
$B_8 + E_3$	48	3.458	3.164	78	3.653	3.132
$B_9 + E_4$	70	3.006	2.709	104	3.343	3.191
$B_{10} + E_5$	60	2.675	2.590	86	3.110	2.941
$C_1 + D_7$	35	3.209	3.046	38	3.565	3.216
$C_2 + D_8$	40	3.793	3.976	51	3.929	3.770
$C_3 + D_9$	68	4.162	3.879	87	4.196	3.705
$C_4 + D_{10}$	43	4.082	4.462	63	4.178	4.441
$C_5 + D_{11}$	38	3.671	3.643	52	3.937	3.918
$C_6 + D_{12}$	32	3.344	3.621	39	3.733	3.956
$C_7 + D_1$	27	3.474	3.135	47	3.790	3.932
$C_8 + D_2$	37	3.872	4.826	54	4.026	4.783
$C_9 + D_3$	46	4.076	4.018	60	4.158	4.339
$C_{10} + D_4$	61	3.849	3.654	75	4.016	4.038
$C_{11} + D_5$	38	3.315	3.210	53	3.676	3.861
$C_{12} + D_6$	34	2.954	3.375	42	3.489	3.386

The table XII gives this comparison for the groups »magn ≤ 4.9» and »all stars». It is however very difficult from this table to draw any positive conclusions to the favour of the ellipsoidal hypothesis. A graphical comparison will meet with obstacle. I have however in the fig. 14 and fig. 15 for the B-squares and the C-squares compared the observed and the recomputed values of the dispersion for the group »magn ≤ 4.9». The former are represented by small circles and the latter by the full drawn line. The agreement seems very good in spite of the small number of stars used. The shortest axis, situated in the square B_{10} is very well marked out by the observations. It is however to be observed that these curves are only sections through the surface in question.

36. From the above results it may be established that the three axial velocity distribution of the stars as derived from radial velocity observations is closely related to the galactic system, and further that the longest and shortest axes of the

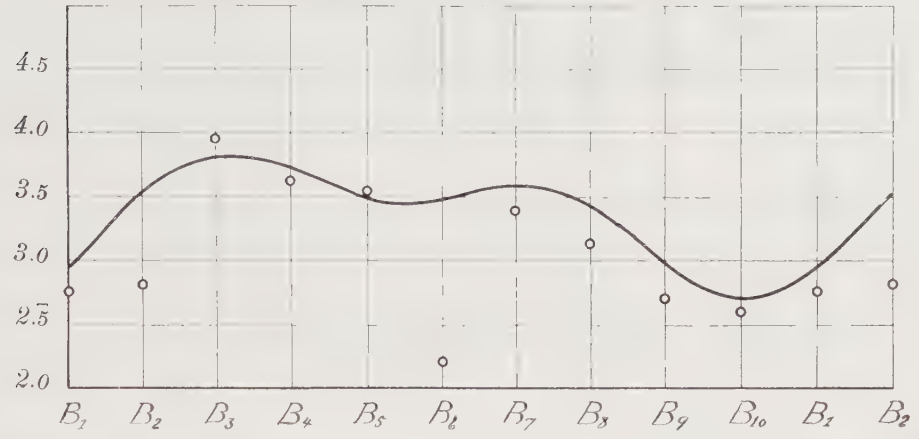

Fig. 14.

Graphical comparison between observed and computed dispersion in the line of sight for stars of magnitudes ≤ 4.9. Squares B_1 to B_{10}.

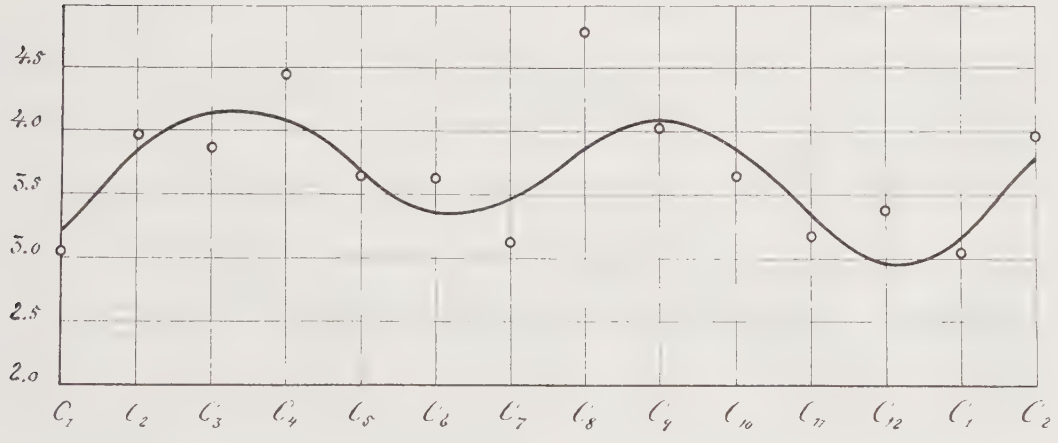

Fig. 15.

Graphical comparison between observed and computed dispersion in the line of sight for stars of magnitudes ≤ 4.9. Squares C_1 to C_{10}.

ellipsoids are both coincident with the plane of the Milky Way. It is true that the directions of the axes of the ellipsoid from the different groups are diverging, but this divergence is surely due to the small number of observations. In each case the arrangement of the three axes is the same.

Examination of the velocity distribution of stars of different spectral classes in relation to the galactic system.

37. The velocity distribution being known to be closely related to the plane of the Milky Way, it may be convenient from many points of wiew directly to determine the moments in the galactic system by a transformation of the coordinates. To this end I have used a galactic system defined in the following way: The positive Z-axis is directed towards the pole of the Milky Way: $\alpha = 191^0.2$, $\delta = +28^0.0$, the positive Ξ-axis is directed towards a point: $\alpha = 274^0.6$, $\delta = -12^0.0$, situated in the Milky Way and approximatively coinciding with the principal vertex. The H-axis lies in the plane of the Milky Way.

For the relations between this system, denoted by G, and the other systems I have used the following denominations and direction cosines

	G:		
	Ξ	H	Z
X	β_{11}	β_{21}	β_{31}
K_3: Y	β_{12}	β_{22}	β_{32}
Z	β_{13}	β_{23}	β_{33}

	K_2:		
	X''	Y''	Z''
Ξ	a_{11}	a_{21}	a_{31}
G: H	a_{12}	a_{22}	a_{32}
Z	a_{13}	a_{23}	a_{33}

The numerical values of the direction cosines I have published in another paper [1]; here I only reprint the a_{ij} quantities, owing to some small misprints in the paper mentioned:

$$\begin{aligned} a_{11} &= +0.0801, & a_{12} &= +0.4933, & a_{13} &= -0.8661, \\ a_{21} &= -0.9749, & a_{22} &= -0.1425, & a_{23} &= -0.1715, \\ a_{31} &= -0.2079, & a_{32} &= +0.8581, & a_{33} &= +0.4695. \end{aligned}$$

38. Let the three velocity components of a single star in the system K_3 be denoted by

$$U, \quad V, \quad W,$$

and let the velocity components of the same star in the system G be denoted by

$$\Xi, \quad \mathrm{H}, \quad \mathrm{Z},$$

then from the schemes above we get the following relations:

$$\begin{aligned} U &= \beta_{11}\,\Xi + \beta_{21}\,\mathrm{H} + \beta_{31}\,\mathrm{Z}, \\ V &= \beta_{12}\,\Xi + \beta_{22}\,\mathrm{H} + \beta_{32}\,\mathrm{Z}, \\ W &= \beta_{13}\,\Xi + \beta_{23}\,\mathrm{H} + \beta_{33}\,\mathrm{Z}, \end{aligned} \tag{36}$$

[1] On the absolute motion of 160 parallax-stars by K. W. Gyllenberg. Meddelande fr. Lunds Astr. Obs. No. **65**.

and

$$
\begin{aligned}
\Xi &= \beta_{11}\, U + \beta_{12}\, V + \beta_{13}\, W, \\
\mathrm{H} &= \beta_{21}\, U + \beta_{22}\, V + \beta_{23}\, W, \\
\mathrm{Z} &= \beta_{31}\, U + \beta_{32}\, V + \beta_{33}\, W.
\end{aligned} \tag{36*}
$$

The moments in the galactic system are now easily found through the transformation of the coordinates.

In general we get:

$$M(U^i V^j W^k) = M\left[(\beta_{11}\Xi+\beta_{21}\mathrm{H}+\beta_{31}\mathrm{Z})^i\,(\beta_{12}\Xi+\beta_{22}\mathrm{H}+\beta_{32}\mathrm{Z})^j\,(\beta_{13}\Xi+\beta_{23}\mathrm{H}+\beta_{33}\mathrm{Z})^k\right]. \tag{37}$$

and

$$M(\Xi^i \mathrm{H}^j \mathrm{Z}^k) = M\left[(\beta_{11}U+\beta_{12}V+\beta_{13}W)^i(\beta_{21}U+\beta_{22}V+\beta_{23}W)^j(\beta_{31}U+\beta_{32}V+\beta_{33}W)^k\right]. \tag{37*}$$

Regarding the first of these expressions for $i=j=0$ and $k=2$, and developing the right member, we now get the following relation:

$$
\begin{aligned}
N_{002} = {} & \beta_{13}^2 N^G_{200} + \beta_{23}^2 N^G_{020} + \beta_{33}^2 N^G_{002} + 2\,\beta_{23}\,\beta_{33} N^G_{011} + 2\,\beta_{13}\,\beta_{33} N^G_{101} + \\
& + 2\,\beta_{13}\,\beta_{23} N^G_{110}.
\end{aligned} \tag{38}
$$

Here are

$$N_{ijk} = M(U^i V^j W^k),$$

and

$$N^G_{ijk} = M(\Xi^i \mathrm{H}^j \mathrm{Z}^k).$$

The left member of (38) is the observed quantity in the square and the right member contains the unknown moments in the galactic system. The number of these may however be reduced. Assuming the velocity distribution uncorrelated in this system, the last three unknown quantities may be put equal to zero. In the same manner we may solve the equation when the frequency surface is assumed to be represented by an ellipsoid of revolution. Then we have to put

$$N^G_{020} = N^G_{002}.$$

Here however the restriction remains that the frequency function is regarded as uncorrelated in the Ξ-H plane as well as the Ξ-Z plane.

From equation (38) the dispersions in the three galactic main directions were now computed. In the table XIII the results are given for each separate spectral-class. The dispersions are indicated by Σ_{ijk}

TABLE XIII.

The mean velocities in the three main directions in the galactic system.

Spectral Class	Number of stars	Σ_{200}	Σ_{020}	Σ_{002}
B	247	1.863 ± 0.549	2.002 ± 0.216	1.268 (± 0.162)
A	263	3.932 ± 0.260	2.338 ± 0.754	2.676 ± 0.519
F	237	4.546 ± 0.338	3.376 ± 0.299	3.306 ± 0.696
G	208	4.921 ± 0.387	2.643 ± 1.063	4.549 ± 0.877
K	486	4.480 ± 0.191	3.979 ± 0.167	4.149 ± 0.382
M	85	4.362 ± 1.029	4.829 ± 2.641	4.437 ± 2.013
magn $\leq$ 4.9	1069	4.178 ± 0.093	2.631 ± 0.244	3.565 ± 0.186

39. With the exception of the class *B* stars, the mean velocities in the three directions agree very well with the results found from the solution of the velocity ellipsoid, where the spectral classes were taken in groups. Concerning the Class *B* stars, their velocity distribution seems in opposition to the other stars to be represented by an ellipsoid, flattened in the plane of the Milky Way. I will however discuss this fact in another place.

The relative magnitude of the mean velocities in the separate spectral classes are the same as before, a fact that is very striking. It seems as if the mean velocity in the direction of the principal vertex, except for the *B* class stars, were almost constant and the mean velocities in the other directions gradually increase when passing through the spectral classes from *A* to *M*. In the *M* class the three axes are equal, the velocity distribution is here almost spherical, quite in agreement with the proposition pronounced by Charlier that the velocity distribution of stars with increasing age must necessarily accept a globular shape.

But how can this circumstance be connected with the theory of giant and dwarf stars, and the existence of two types of age in each spectral class? Especially concerning the class *M* stars there exists a contradiction. These stars necessarily contain a majority of giant stars, at least when, as here, only the bright stars are considered. Is this really the case the globular velocity surface of the class *M* stars will represent the distribution of the velocities of the youngest of the stars.

A peculiarity that, as mentioned, seems quite striking, is the fact that the velocity ellipsoids have their shortest and longest axes situated in the plane of the Milky Way. The velocity ellipsoids are flattened perpendicular to this plane and not flattened along it, as was found from proper motion observations. It is however not difficult to overcome this contradiction as far as the proper motion results are concerned. A variation in terms of galactic latitude of the mean parallaxes of the

stars or perhaps also the parameter λ_1 will be sufficient to bring the two ellipsoids to coincide.

40. As remarked before, the class B stars showed a very singular velocity distribution. CAMPBELL says that for these stars there seems to exist no distinct vertex direction, further he points out that the average velocity of the same stars in a region of the sky $\alpha = 12^h$ to 18^h and $\delta = 0^0$ to -70^0 seems to be smaller than for the other parts of the sky.

The equation (38) however gives us a possibility to study this case more closely when all moments of the right membrum are considered.

In spite of the small total number of stars, which will cause large mean errors in the six unknown quantities, the complete solution of (38) will give an idea of the position of the velocity ellipsoid. The solution gave the following values of the moments:

$$N^G_{200} = +3.979,\quad N^G_{020} = +4.084,\quad N^G_{002} = +2.438,$$
$$N^G_{011} = +0.211,\quad N^G_{101} = -2.143,\quad N^G_{110} = +0.167.$$

From these values the position of the velocity ellipsoid and the length of its axes were computed. This ellipsoid was found to be represented by a surface of revolution. The two equal axes have a length of 2.166 Sm. and the third — the shortest — has a length of 0.956 Sm. and is directed towards a point: $\alpha = 244^0.6$, $\delta = +12^0.2$, or about 35^0 from the pole of the Milky Way. This point lies in the square C_8. A mean of the dispersion in this and the surrounding squares gave $\sigma = 1.406 \pm 0.176$ Sm. from 32 stars.

Now it may be observed that the distribution of Class B stars relative to the galactic plane will cause a large uncertainty in the determination of the axis directed towards the pole of the Milky Way. I found it therefore convenient to make a new solution assuming the moments

$$N^G_{011} = N^G_{101} = 0.$$

This solution gave the following results:

$$N^G_{200} = +3.486,\quad N^G_{020} = +4.008,\quad N^G_{002} = +1.604,\quad N^G_{110} = +0.082.$$

The last moment shows a very small correlation. The moment N^G_{020} seems to point to a large mean velocity in this direction, but, as the mean error is large, the velocity distribution is quite circular in the plane of the Milky Way. To test the reality of the flat shape of the ellipsoid I have computed the mean velocities for the two groups of squares surrounding the plane and the pole of the Milky Way.

The following results were obtained:

	Class B stars surrounding	
	the plane of the M. W.	the pole of the M. W.
Number of stars	188	59
Mean velocity	$1{,}936 \pm 0.100$	1.555 ± 0.143
Values computed from the velocity ellipsoid constants	1.867 2.002	1.267

The difference seems to indicate the reality of a peculiar velocity distribution of the class B stars. However, if there exists a difference in the value of K in different parts of the sky this circumstance will largely influence the results.

41. It seems very remarkable that the stars of type B have like the stars of type, M, a circular distribution in the plane of the Milky Way. This is unexpected and we should indeed expect to find the largest divergence between these two groups. Regarding however the proportion of giant and dwarf stars in these spectral classes, a certain similarity is to be found, which perpaps will help to explain the fact. The class Bstars, having small dispersion of the absolute magnitude, contain exclusevily stars of the giant type. In the following classes the relative number of entering dwarf stars increases and has its maximum in one of the intervening classes. As the material may approximatively be considered to contain stars brighter than a certain magnitude the giant stars belonging to the class M will be relatively favoured, while only the nearest of the dwarf stars enter. The catalogue contains further only seven stars of type M, whose parallaxes are determined. Also, the determination of the mean parallax of the class M stars gives a small value pointing to the presence of large number giant stars.

Returning to the examination of the constant K, this was shown to be almost equal in sign and magnitude for stars of classes B and M. This was found to be also the case both for the sun's relative velocity as for the mean distances. And finally looking back to the figures 2—8 we found a new similarity concerning the variation of the constant K in terms of angular distances from the principal vertex. The two spectral types thus give a distinct positive displacement of the mean in the direction of the principal vertices, without giving any tendency to a larger mean velocity in the same directions.

Unfortunately there are too few stars to confirm any explanations.

The mean errors in the moments of the second order.

42. The quantities in table XIII are computed by a linear transformation of the observed values of σ. Let the mean error in one observation of the left membrum of (38) be

$$\sigma'' = \varepsilon(\sigma^2),$$

then the mean errors in the unknown quantities are expressed through the formula

$$\varepsilon(N_{ijk}^{g}) = \frac{\sigma''}{\sqrt{p}}.$$

Observing that

$$\varepsilon(\sigma^2) = 2\,\sigma\varepsilon(\sigma)$$

and

$$\varepsilon(\sigma) = \frac{\sigma}{\sqrt{2n}}.$$

where n is equal to the number of stars in a certain square. Putting now for n a mean number so that

$$n = N : 24$$

and writing for σ and $\varepsilon(\sigma)$ the values from table X, then the formula takes the form

$$\varepsilon(\sigma^2) = 2\,\sigma\,\varepsilon(\sigma)\sqrt{24}$$

and consequently

$$\varepsilon(N^{ij}_{200}) = \frac{2\,\sigma\,\varepsilon(\sigma)\sqrt{24}}{\sqrt{p}}.$$

A new approximation by putting $\Sigma_{ijk} = \sigma$ in the different cases will give the final expression:

$$\varepsilon(\Sigma_{ijk}) = \frac{\varepsilon(\sigma)\sqrt{24}}{\sqrt{p}}.$$

In this way the mean errors in table XIII were computed. Especially for the class B stars they will be erroneous, but the values on the whole give a view of the magnitude of the mean errors for each spectral group.

Concerning the mean error in the determination of the velocity ellipsoids, I have for the group »stars of magn. $\leq 4{,}9$» derived the mean errors in the lengths of the axes. These were found to be equal for the three axes and amounting to 0,160 Sm. From this value I have with the aid of the short method, used by Mr. Wicksell, computed the mean error in the galactic coordinates of the vertices. The following values were obtained:

	Ξ-axis	H-axis
Mean error in gal. latitude	$\pm 4^0.3$,	$\pm 4^0.3$,
» » » » longitude	$\pm 3^0.4$,	$\pm 17^0.2$.

For the position of the axis directed towards the pole of the Milky Way, the mean error in the direction Ξ — Z is equal to $\pm 3^0.4$, and in the direction H — Z the mean error amounts to $\pm 17^0.2$.

The moments of higher order.

43. In the following table XIV the values of the skewness — S — and the excess — E — are tabulated. Owing to the small number of stars I have combined the spectral classes in groups; the stars of classes B and A were however separated on account of the peculiar velocity distribution of the former class.

For each computation the stars from six representative squares were used. Further the stars were limited to magn ≤ 4.9.

Unfortunately the number of stars in the different spectral groups are small. In several cases the mean errors are too large to give a real meaning of the characteristic. The throughout negative skewness in the direction of the principal vertex is

TABLE XIV.

The values of the skewness — S — and the excess — E — in the three main directions of the galactic system.

The skewness.

Sp. Group	Ξ		H		Z	
	N	S	N	S	N	S
B	62	+ 0.081	82	— 0.374	—	—
A	57	— 0.141	42	— 0.128	50	+ 0.016
F and G	67	— 0.083	62	+ 0.039	45	+ 0.059
K and M	109	— 0.159	118	+ 0.068	90	+ 0.013
magn. $\leq$ 4.9	295	— 0.134	304	+ 0.007	196	+ 0.008

The excess.

Sp. Group.	Ξ		H		Z	
	N	E	N	E	N	E
B	62	+ 0.014	82	+ 0.181	—	—
A	57	— 0.114	42	± 0.000	50	— 0 016
F and G	67	— 0.053	62	— 0.026	45	+ 0.192
K and M	109	— 0.024	118	+ 0.075	90	+ 0.021
magn. $\leq$ 4.9	295	+ 0.004	304	+ 0.109	196	+ 0.116

remarkable. Further the large difference between the values of S for the class B stars in the Ξ and H directions is very striking.

The most interesting of these values are perhaps the results from the group magn. $\leq$ 4.9, since for this group it is possible to get a comparison with the same characteristics found from proper motion observations.

Recently Mr. Wicksell has finished an investigation of the proper motions of the stars of magnitude brighter than 6.0, in which he gives the characteristics up to the fourth order of the frequency function of the *linear* motions referred to a system of coordinates very nearly coincident with the galactic system used in this chapter and denoted by G.

From his tables, which he kindly lent me, it appears that the skewness and the excess of the motions as projected on the Ξ, H and Z axes are as given in the following cheme. To get a comparison I reprint the values from table XIV adding the mean errors.

Graphical representation of the frequency function of the velocities of stars of magn ≤ 4.9 in the three main directions of the Milky Way.

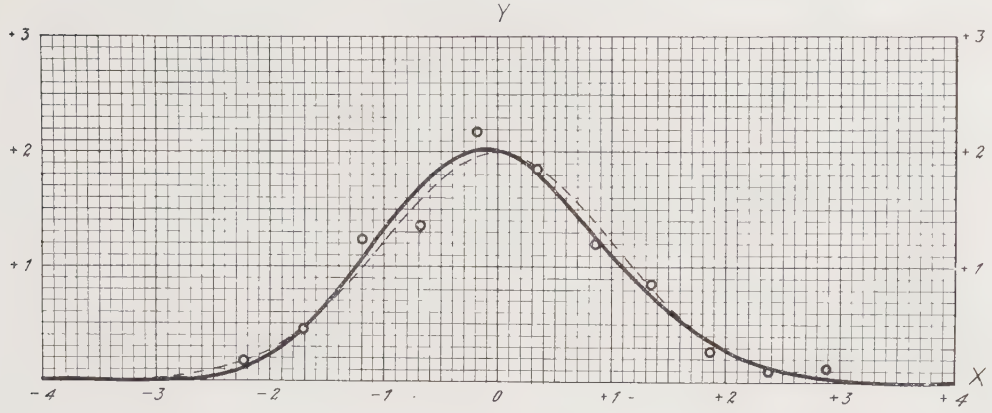

Fig. 16. Ξ *Principal vertex*

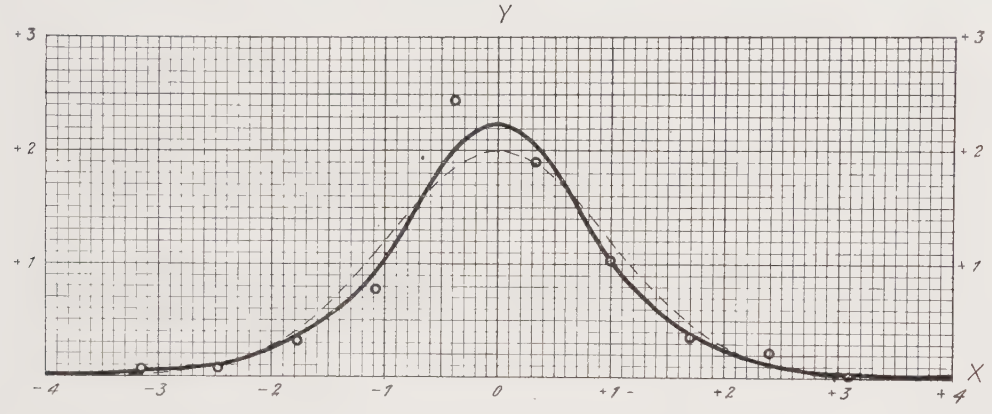

Fig. 17. H

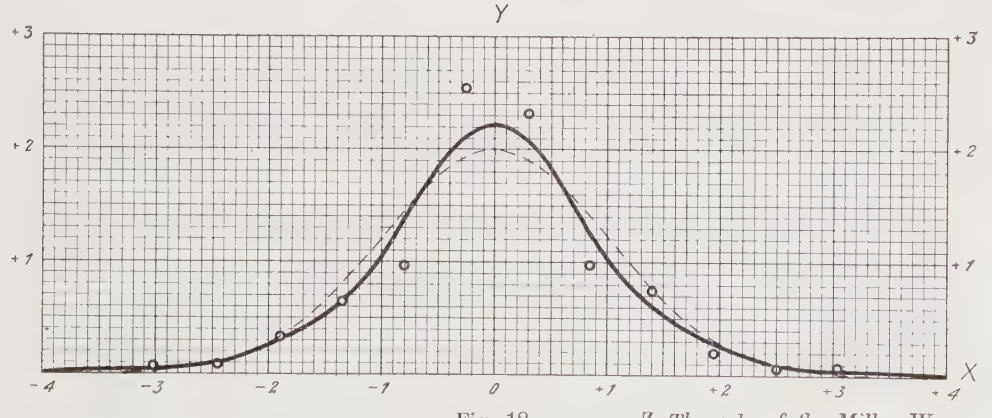

Fig. 18. Z *The pole of the Milky Way*

		Values from radial velocities (Gyllenberg) stars of magn. $\leq 4.^{9}$.	Values from proper motions (Wicksell) stars of magn. $\leq 5.^{9}$.
Skewness:	Ξ	-0.134 ± 0.113	-0.084 ± 0.017
	H	$+0.007 \pm 0.111$	$+0.039 \pm 0.078$
	Z	$+0.008 \pm 0.138$	$+0.131 \pm 0.038$
Excess:	Ξ	$+0.004 \pm 0.036$	-0.141 ± 0.007
	H	$+0.109 \pm 0.035$	-0.028 ± 0.033
	Z	$+0.116 \pm 0.044$	$+0.273 \pm 0.085$

Except the in case of the excess in the Ξ components and perhaps also in the H components these values are of the same order of magnitude as ours for stars of magn. $\leq 4.^{9}$. As pointed out by Wicksell another choice of the numerical value of the fundamental stellar constant q', entering the density and luminosity functions, on which his results depend, would make his values grow considerably in magnitude and make them still less in consistence with the above radial velocity results.

In the figures 16—18 I have made a graphical representation of the frequency function of the velocities when regard is taken to the higher characteristics. The dotted line denotes the normal curve. The small circles represent the observations and the drawn line is the theoretical curve.

CHAPTER IV.

A treatment of proper motion and radial velocity in common

44. As mentioned in the first chapter the proper motions in right ascension and declination were in the catalogue noted for each star. In this chapter I will through a combination of proper motion and radial velocity observations derive the mean parallaxes of the stars.

Before going further, however, I have rejected all stars whose proper motions exceed 0″.500 per year. In his memoir CHARLIER rejects all stars having proper motions larger than 0″.400. I have altered this limit because it seems to me that too many stars will be rejected in this way, compared with those having radial velocities larger than 14.0 Sm.

The remaining stars are distributed in the squares according to the table XV. Here is also noted the number of rejected stars.

TABLE XV.

Number of stars after rejecting those whose velocities in any direction are very large.

Square	n	x or $y > 0.500$ $z > 1.400$
A_1	16	1
A_2	25	5
B_1	31	10
B_2	31	6
B_3	24	2
B_4	22	6
B_5	24	8
B_6	23	1
B_7	22	5
B_8	31	4
B_9	39	4
B_{10}	31*	3
C_1	21	3
C_2	26	4
C_3	49	4
C_4	26*	7
C_5	22	1
C_6	19	1
C_7	28	2
C_8	26	5
C_9	18	4
C_{10}	33	8
C_{11}	27	3
C_{12}	27	3

Square	n	x or $y > 0.500$ $z > 1.400$
D_1	15	2
D_2	19	4
D_3	36*	4
D_4	35*	3
D_5	23	1
D_6	12	—
D_7	13	2
D_8	22	4
D_9	34	2
D_{10}	30	1
D_{11}	27	2
D_{12}	19	—
E_1	32	5
E_2	36	3
E_3	43	3
E_4	62	2
E_5	52	1
E_6	49	1
E_7	46*	3
E_8	43	2
E_9	32	5
E_{10}	32	3
F_1	36	2
F_2	28	2

* Including one star of type *Oe*.

General formulæ for determining the apex direction and related constants.

45. Let, as before, the relative velocity of the sun in the system K_2 be expressed through the three components

$$U_0'', \quad V_0'', \quad W_0''.$$

Assume further the displacement of the mean in each square to be caused by the sun's motion, then these displacements

$$x_0, \quad y_0, \quad z_0,$$

may be expressed in linear measure through

$$\begin{aligned} x_0 &= \vartheta_1 X_0, \\ y_0 &= \vartheta_1 Y_0, \\ z_0 &= \quad Z_0. \end{aligned} \tag{39}$$

Here ϑ_1 denotes the mean of the reciprocal value of the distances, so that

$$\vartheta_1 = M\left(\frac{1}{r}\right). \tag{40}$$

The third coordinate, the mean from the radial velocity observations may be considered as indepedent of the distances.

Knowing the three volocity components in the K_3 system, we may according to the scheme page 13 express them in relation to the system K_2 so that

$$\begin{aligned} \gamma_{11}\vartheta_1 U_0'' + \gamma_{21}\vartheta_1 V_0'' + \gamma_{31}\vartheta_1 W_0'' - x_0 &= 0, \\ \gamma_{12}\vartheta_1 U_0'' + \gamma_{22}\vartheta_1 V_0'' + \gamma_{32}\vartheta_1 W_0'' - y_0 &= 0, \\ \gamma_{13} \quad U_0'' + \gamma_{23} \quad V_0'' + \gamma_{33} \quad W_0'' - z_0 &= 0. \end{aligned} \tag{41}$$

46. It is however possible to generalize these formulæ.

At first we may assume that the node of the invariable plane has a retrograde motion in relation to a fixed plane [1]. Charlier has deduced the necessary expressions, but owing to the new definition of the positive axes I will here reprint the complete formula.

Let the velocity of rotation about the instantaneous axis be ω, then we have

$$\begin{aligned} \omega_x &= \omega \cos\delta_0 \cos\alpha_0, \\ \omega_y &= \omega \cos\delta_0 \sin\alpha_0, \\ \omega_z &= \omega \sin\delta_0 \quad . \end{aligned} \tag{42}$$

---

[1] Se Charlier loc. cit. pages 76—78.

The variations caused by this rotation in the angular coordinates of a star now get the form:

$$\begin{aligned} \cos\delta\,\Delta\alpha &= \sin\delta\,\cos\alpha\,\omega_x + \sin\delta\,\sin\alpha\,\omega_y - \cos\delta\,\omega_z\,, \\ \Delta\delta &= -\sin\alpha\,\omega_x + \cos\alpha\,\omega_y\,. \end{aligned} \tag{43}$$

Considering however the expression for the direction cosines

$$\begin{aligned} \gamma_{12} &= -\sin\delta\cos\alpha, \\ \gamma_{22} &= -\sin\delta\sin\alpha, \\ \gamma_{32} &= \cos\delta\,, \\ \gamma_{11} &= -\sin\alpha\,, \\ \gamma_{21} &= \cos\alpha\,, \end{aligned}$$

the formulæ (44) may be written

$$\begin{aligned} \cos\delta\,\Delta\alpha &= -\gamma_{12}\,\omega_x - \gamma_{22}\,\omega_y - \gamma_{32}\,\omega_z\,, \\ \Delta\delta &= \gamma_{11}\,\omega_x + \gamma_{21}\,\omega_y. \end{aligned} \tag{44}$$

The quantities ω_x, ω_y and ω_z being known, the formula (42) gives the position of the axis of rotation as well as the annual amount of this rotation.

Assume further the observed means of the radial velocities to be affected by a displacement — K — of objective existence or due to an error in the measurements, the final equations get the following forms:

$$\begin{aligned} &\gamma_{11}\vartheta_1\,U_0'' + \gamma_{21}\vartheta_1\,V_0'' + \gamma_{31}\vartheta_1\,W_0'' - \gamma_{12}\,\omega_x - \gamma_{22}\,\omega_y - \gamma_{32}\,\omega_z - x_0 = 0, \\ &\gamma_{12}\vartheta_1\,U_0'' + \gamma_{22}\vartheta_1\,V_0'' + \gamma_{32}\vartheta_1\,W_0'' + \gamma_{11}\,\omega_x + \gamma_{21}\,\omega_y - y_0 = 0, \\ &\gamma_{13}\,U_0'' + \gamma_{23}\,V_0'' + \gamma_{33}\,W_0'' + K - z_0 = 0. \end{aligned} \tag{45}$$

47. Forming the normal equations, we get from each of the equations (45) three different systems:

a. Normal equations from the motion in right ascension:

$$\begin{aligned} &[\gamma_{11}\gamma_{11}]\,\vartheta_1\,U_0 - [\gamma_{11}x_0] = 0, \\ &[\gamma_{21}\gamma_{21}]\,\vartheta_1\,V_0 - [\gamma_{21}x_0] = 0, \\ &\quad\cdots\qquad\cdots\qquad\cdots \\ &[\gamma_{12}\gamma_{12}]\,\omega_x + [\gamma_{12}x_0] = 0, \\ &[\gamma_{22}\gamma_{22}]\,\omega_y + [\gamma_{22}x_0] = 0, \\ &[\gamma_{32}\gamma_{32}]\,\omega_z + [\gamma_{32}x_0] = 0. \end{aligned} \tag{46}$$

b. Normal equations from the motion in declination:

$$\begin{aligned} &[\gamma_{12}\gamma_{12}]\,\vartheta_1\,U_0'' - [\gamma_{12}y_0] = 0, \\ &[\gamma_{22}\gamma_{22}]\,\vartheta_1\,V_0'' - [\gamma_{22}y_0] = 0, \\ &[\gamma_{32}\gamma_{32}]\,\vartheta_1\,W_0'' - [\gamma_{32}y_0] = 0, \\ &\quad\cdots\qquad\cdots\qquad\cdots \\ &[\gamma_{11}\gamma_{11}]\,\omega_x - [\gamma_{11}y_0] = 0, \\ &[\gamma_{21}\gamma_{21}]\,\omega_y - [\gamma_{21}y_0] = 0, \\ &\quad\cdots\qquad\cdots\qquad\cdots \end{aligned} \tag{47}$$

c. Normal equations from the motion in the line of sight:

$$
\begin{aligned}
&[\gamma_{13}\,\gamma_{13}]\;U_0'' - [\gamma_{13}\,z_0] = 0,\\
&[\gamma_{23}\,\gamma_{23}]\;V_0'' - [\gamma_{23}\,z_0] = 0,\\
&[\gamma_{33}\,\gamma_{33}]\;W_0'' - [\gamma_{33}\,z_0] = 0,\\
&\qquad\qquad 48K - [z_0] = 0.
\end{aligned}
\tag{48}
$$

In the last equation the coefficient of K denotes the number of squares. The equations (48) are indeed identical with (5) chapter II.

Solving the three systems simultaneously, we may multiply the last system with ϑ_1. Then we get

$$
\begin{aligned}
&48\,\vartheta_1\;U_0'' - [\gamma_{11}\,x_0] - [\gamma_{12}\,y_0] - [\gamma_{13}\,z_0]\,\vartheta_1 = 0,\\
&48\,\vartheta_1\;V_0'' - [\gamma_{21}\,x_0] - [\gamma_{22}\,y_0] - [\gamma_{23}\,z_0]\,\vartheta_1 = 0,\\
&48\,\vartheta_1\;W_0'' \qquad\quad - [\gamma_{32}\,y_0] - [\gamma_{33}\,z_0]\,\vartheta_1 = 0,
\end{aligned}
\tag{49}
$$

and

$$
\begin{aligned}
&[\gamma_{12}\,\gamma_{12} + \gamma_{11}\,\gamma_{11}]\,\omega_x + [\gamma_{12}\,x_0] - [\gamma_{11}\,y_0] = 0,\\
&[\gamma_{22}\,\gamma_{22} + \gamma_{21}\,\gamma_{21}]\,\omega_y + [\gamma_{22}\,x_0] - [\gamma_{21}\,y_0] = 0,\\
&\qquad [\gamma_{32}\,\gamma_{32}]\,\omega_z + [\gamma_{32}\,x_0] \qquad\qquad = 0,\\
&\qquad\qquad 48K - [z_0] = 0.
\end{aligned}
\tag{49*}
$$

48. As seen from what precedes the solution of the motion of the invariable plane as well as of the constant K is quite independent of the solution of the apex direction and the solar motion. This will however only occur when equal weights are given to all squares.

If the weights however vary in any way or if the stars are treated separately the equations get the form:

$$
\begin{aligned}
&P\,\vartheta_1\;U_0'' - [\gamma_{11}\,px_0] - [\gamma_{12}\,py_0] - [\gamma_{13}\,pz_0]\,\vartheta_1 = 0,\\
&P\,\vartheta_1\;V_0'' - [\gamma_{21}\,px_0] - [\gamma_{22}\,py_0] - [\gamma_{23}\,pz_0]\,\vartheta_1 = 0,\\
&P\,\vartheta_1\;W_0'' \qquad\quad - [\gamma_{32}\,py_0] - [\gamma_{33}\,pz_0]\,\vartheta_1 = 0,
\end{aligned}
\tag{50}
$$

Here p denotes the weight in a single square and

$$P = \Sigma\, p_i \qquad\qquad (i = 1,\ 2\ \ldots\ 48)$$

The weights being assumed equal to the numbers of stars we get

$$p = n, \qquad P = N,$$

where n is the number of stars in a single square and N the total number.

In the formula (50) the unknown quantity K is rejected in the present solution because the velocities are already corrected for this constant.

49. The formula (50) may, however, be obtained in a direct way. Observing that

$$\begin{aligned} X'' &= \gamma_{11}\, x + \gamma_{12}\, y + \gamma_{13}\, z, \\ Y'' &= \gamma_{21}\, x + \gamma_{22}\, y + \gamma_{23}\, z, \\ Z'' &= \gamma_{31}\, x + \gamma_{32}\, y + \gamma_{33}\, z, \end{aligned}$$

we get by summing up:

$$\begin{aligned} \Sigma\, X'' &= N\, \vartheta_1\ U_0'' = [\gamma_{11}\, \Sigma x] + [\gamma_{12}\, \Sigma y] + [\gamma_{13}\, \Sigma z]\, \vartheta_1, \\ \Sigma\, Y'' &= N\, \vartheta_1\ V_0'' = [\gamma_{21}\, \Sigma x] + [\gamma_{22}\, \Sigma y] + [\gamma_{23}\, \Sigma z]\, \vartheta_1, \\ \Sigma\, Z'' &= N\, \vartheta_1\ W_0'' = [\gamma_{31}\, \Sigma x] + [\gamma_{32}\, \Sigma y] + [\gamma_{33}\, \Sigma z]\, \vartheta_1. \end{aligned}$$

Observing that

$$\Sigma x = n x_0,$$

these formulae are found to be quite identical with (50).

50. In the following all stars fainter than 5.0 were rejected. Such a limitation was made to get results comparable with the proper motion results.

In table XVI the mean displacements for each spectral group in proper motions and radial velocities are tabulated. The table XVII gives the same values for stars brighter than 4.9.

From the equation (49*) the motion of the invariable plane was obtained and expressed through

$$\begin{aligned} \omega_x &= -\ 0.00287, \\ \omega_y &= -\ 0.00165, \\ \omega_z &= +\ 0.00234, \end{aligned}$$

wich gives

$$\begin{aligned} \omega &= +\ 0.00406, \\ \alpha_0 &= \ \ 209^0.3, \\ \delta_0 &= +\ 30^0.2. \end{aligned}$$

Thus the invariable plane has a direct motion to the amount of 0.00406 seconds of arc per year, which value agrees very well with that found by Charlier. The axis of rotation is directed towards a point about 15 degrees from the pole of the Milky Way. For the same motion Charlier has found:

$$\begin{aligned} \omega &= +\ 0.003528, \\ \alpha_0 &= \ \ 186^0.7, \\ \delta_0 &= +\ 15^0.1. \end{aligned}$$

51. From the equations (49) and (50) the components of the sun's relative velocity were now computed. Considering these velocities to be known from table IV we may easily find the corresponding value of ϑ_1.

TABLE XVI.

The mean displacements in proper motion and radial velocity of stars of magn. ≤ 4.9.

Square	Class *B*				Class *A*				Class *F*			
	n	x_0	y_0	z_0	n	x_0	y_0	z_0	n	x_0	y_0	z_0
A_1	—	—	—	—	2	— 0.021	+ 0.031	— 0.11	2	+ 0.035	— 0.062	— 1.99
A_2	2	0.	+ 0.020	— 3.65	1	— 0.002	+ 0.017	— 1.69	1	— 0.003	— 0.268	— 2.20
B_1	8	+ 0.022	— 0.010	— 1.45	5	+ 0.158	— 0.021	+ 1.40	1	— 0.071	+ 0.174	+ 4.50
B_2	9	+ 0.012	— 0 011	+ 0.41	—	—	—	—	4	+ 0.056	— 0.040	+ 0.81
B_3	1	+ 0.031	— 0.075	— 0·28	5	0.	— 0.023	+ 0.08	1	+ 0.006	— 0.013	+ 0.80
B_4	—	—	—	—	8	— 0.090	— 0.056	+ 0.59	4	— 0.058	— 0.107	+ 0.89
B_5	—	—	—	—	7	+ 0.007	— 0.018	— 1.37	3	— 0.204	— 0.068	+ 3.93
B_6	1	— 0.119	— 0.021	— 2.18	11	— 0 022	+ 0.009	— 2.05	2	— 0.069	— 0.215	— 1.89
B_7	2	+ 0.022	— 0.019	— 1.85	3	— 0.030	+ 0.009	— 1.63	4	— 0.051	+ 0.165	— 2.88
B_8	5	— 0.007	— 0 001	— 3.82	7	+ 0.036	+ 0.058	— 3.52	2	+ 0.126	+ 0.027	— 5.03
B_9	7	+ 0.003	— 0.005	— 5.30	3	+ 0.022	+ 0.007	— 2.43	3	— 0.006	+ 0.080	— 3.60
B_{10}	7	+ 0.007	— 0.005	— 3.50	5	+ 0.123	+ 0.006	— 0.46	3	+ 0.025	+ 0.026	— 2.56
C_1	1	0.	— 0.013	+ 0.37	6	+ 0.048	— 0.068	+ 0.36	1	+ 0.017	— 0.232	— 1.48
C_2	8	+ 0.015	— 0 039	+ 1.18	3	+ 0.107	— 0.013	+ 1.93	—	—	—	—
C_3	12	+ 0.006	— 0.031	+ 2.98	11	+ 0.060	— 0.036	+ 6.85	3	+ 0.096	— 0.028	+ 3.10
C_4	—	—	—	—	4	— 0.003	— 0.027	+ 0.71	1	— 0.114	— 0.201	+ 5.89
C_5	—	—	—	—	3	— 0.027	— 0.042	+ 2.11	2	— 0.167	— 0.047	+ 6.84
C_6	—	—	—	—	6	— 0.076	— 0.059	— 0.30	4	— 0.081	— 0.042	— 1.05
C_7	—	—	—	—	3	— 0.003	— 0.016	— 2.53	1	— 0.487	+ 0.026	— 3.25
C_8	—	—	—	—	5	+ 0.049	— 0 012	— 2.57	2	— 0.099	+ 0.002	— 1.22
C_9	1	+ 0.002	— 0.014	— 1.75	3	+ 0.024	— 0.082	— 3.52	2	— 0.107	+ 0.041	— 8.87
C_{10}	4	+ 0.001	— 0.007	— 3.46	5	+ 0.011	+ 0.030	— 4.80	2	+ 0.116	— 0.134	+ 0.43
C_{11}	3	+ 0.023	— 0.012	— 4.29	4	+ 0.033	— 0.034	— 2.25	6	+ 0.088	— 0.125	— 1.50
C_{12}	3	+ 0.007	— 0.002	+ 0.38	5	+ 0.005	— 0.067	— 0.29	5	+ 0.248	— 0.200	— 0.72
D_1	1	+ 0.008	+ 0.001	+ 1.41	—	—	—	—	1	— 0.159	— 0.079	+ 0.21
D_2	2	+ 0.002	— 0.007	+ 1.52	2	+ 0.011	+ 0 006	+ 3.06	2	+ 0.122	— 0.035	— 0.37
D_3	14	+ 0.001	— 0.003	+ 3.88	4	+ 0.003	— 0.018	+ 3.30	4	— 0.078	— 0.040	+ 1.64
D_4	12	— 0.006	+ 0.001	+ 5.87	3	+ 0.005	+ 0.002	+ 7.40	4	— 0.014	+ 0.023	+ 6.10
D_5	—	—	—	—	3	— 0.007	— 0.051	+ 1.48	2	+ 0.019	+ 0.013	+ 6.09
D_6	—	—	—	—	3	— 0.048	— 0.031	+ 2.75	—	—	—	—
D_7	2	— 0.101	— 0.013	— 1.48	2	— 0.051	— 0.034	+ 0.30	2	— 0.181	— 0.058	+ 0.76
D_8	2	— 0.056	— 0.030	— 1.94	4	— 0.078	— 0.075	— 3.59	3	+ 0.009	— 0.261	— 1.15
D_9	5	0.	— 0.032	— 1.81	8	— 0.022	— 0.026	— 3.38	8	+ 0.035	— 0.091	— 0.88
D_{10}	5	+ 0.015	— 0.023	— 2.34	—	—	—	—	4	+ 0.006	— 0.015	— 4.57
D_{11}	—	—	—	—	6	+ 0.036	— 0.030	— 2.47	3	+ 0.059	— 0.072	— 0.81
D_{12}	—	—	—	—	3	+ 0.025	— 0.014	+ 2.25	1	+ 0.207	+ 0.045	+ 5.72
E_1	2	+ 0.051	— 0.038	+ 2.26	2	+ 0.041	— 0.024	+ 0.11	1	+ 0 242	+ 0.044	— 1.06
E_2	2	+ 0.027	+ 0.002	+ 3.04	2	— 0.003	+ 0.016	+ 4.54	3	+ 0.106	+ 0.153	+ 2.83
E_3	7	— 0.006	+ 0.004	+ 3.65	1	— 0.014	— 0.009	+ 5.28	4	— 0.046	+ 0.055	+ 1.49
E_4	12	— 0.022	+ 0.006	+ 3.42	7	— 0.029	— 0.005	+ 3.75	4	— 0.061	— 0.068	+ 4.03
E_5	8	— 0.035	0.	+ 1.53	3	— 0.078	+ 0.017	+ 1.62	6	— 0.036	— 0.008	+ 1.40
E_6	21	— 0.033	— 0.026	+ 1.39	3	— 0.156	— 0.049	— 0.56	4	— 0.118	— 0.065	+ 0.18
E_7	11	— 0.022	— 0.028	— 0 64	2	— 0.048	— 0.066	— 0.62	2	— 0.190	— 0.251	+ 0.26
E_8	8	— 0.006	— 0.040	— 0.40	5	— 0.010	— 0.096	— 1.17	4	+ 0.035	— 0.121	— 2.50
E_9	4	+ 0.012	— 0.069	— 0.40	3	+ 0.086	— 0.036	— 0.28	1	+ 0·108	— 0.063	+ 5.07
E_{10}	1	+ 0.090	+ 0.011	— 0.42	6	+ 0.087	— 0.055	+ 2.17	1	— 0.046	— 0.034	+ 2.11
F_1	3	+ 0.011	+ 0.044	+ 2.89	3	— 0.021	+ 0.075	— 0.23	4	+ 0.018	+ 0.015	+ 1.73
F_2	1	+ 0.011	+ 0.041	+ 1.94	2	— 0.015	— 0.055	+ 0.99	1	— 0.018	+ 0.060	+ 6.12

TABLE XVI (continued).

Square	Class G				Class K				Class M			
	n	x_0	y_0	z_0	n	x_0	y_0	z_0	n	x_0	y_0	z_0
A_1	1	− 0.091	+ 0.003	− 5.76	3	+ 0.035	− 0.035	− 2.62	1	+ 0.014	− 0.043	+ 0.35
A_2	2	+ 0.021	− 0.001	− 3.37	10	− 0.016	+ 0.056	− 2.56	—	—	—	—
B_1	2	− 0.107	− 0.197	− 2.27	6	+ 0.039	− 0.057	− 2.28	1	+ 0 183	− 0.115	− 0.69
B_2	6	+ 0.055	− 0.025	+ 1.83	5	+ 0.047	− 0.047	+ 1.89	2	+ 0.065	− 0.054	+ 1.60
B_3	2	+ 0.045	− 0.221	+ 3 04	8	+ 0.016	− 0.070	+ 0.15	1	+ 0.009	− 0.008	− 0.84
B_4	3	− 0.048	− 0.035	+ 1.86	3	− 0.091	− 0.026	+ 4.71	—	—	—	—
B_5	1	+ 0.006	− 0.003	− 1.33	6	− 0.060	− 0.080	− 0.86	1	− 0.080	+ 0.020	− 5.97
B_6	1	− 0.016	− 0.004	− 5.07	—	—	—	—	2	− 0.011	− 0.008	− 4.00
B_7	5	− 0.120	+ 0.092	− 4.22	4	+ 0.005	− 0.026	− 2 20	2	− 0.045	− 0.166	− 2.42
B_8	1	− 0.014	+ 0.037	− 4.33	6	+ 0.021	+ 0.018	− 5.65	2	+ 0.011	+ 0.089	− 6.76
B_9	3	− 0.026	− 0.069	− 2.98	10	+ 0.041	+ 0.028	− 3.98	—	—	—	—
B_{10}	—	—	—	—	9	+ 0.044	− 0.063	− 3.28	—	—	—	—
C_1	4	− 0.007	+ 0.007	+ 1.50	7	+ 0.014	− 0.024	+ 1.71	1	+ 0.093	− 0.011	+ 8.37
C_2	3	− 0.109	− 0.189	+ 0.42	6	+ 0.100	− 0.062	+ 0.06	1	− 0.014	− 0.077	− 6.41
C_3	3	+ 0.036	− 0.057	+ 6.59	7	+ 0.044	− 0.053	+ 6.15	2	+ 0.019	− 0.001	+ 3.64
C_4	2	− 0.011	− 0.041	+ 3.16	8	− 0.025	− 0.103	+ 2.22	2	− 0.001	− 0.065	+ 6.72
C_5	3	− 0.075	− 0.046	+ 4.67	7	− 0.066	− 0.067	+ 4.16	1	− 0.034	− 0.027	+ 3.81
C_6	—	—	—	—	2	+ 0.104	− 0.111	− 0.14	2	− 0.017	− 0.099	+ 5.98
C_7	3	− 0.097	− 0.114	− 3.14	5	− 0 110	−0. 009	− 0.16	1	− 0.475	− 0.064	− 4.83
C_8	2	− 0.155	− 0.073	− 8.80	11	− 0.014	− 0.035	− 3.65	—	—	—	—
C_9	—	—	—	—	8	− 0.066	+ 0.001	− 5.12	1	− 0.012	+ 0.027	− 7.91
C_{10}	5	+ 0.019	− 0.018	− 2.55	10	+ 0.045	− 0.095	− 6.40	1	+ 0.001	+ 0.009	− 0.48
C_{11}	1	− 0.033	− 0.204	− 1.29	5	− 0.016	− 0.030	− 0.61	1	+ 0.017	− 0.004	− 4.80
C_{12}	2	− 0.061	− 0.039	+ 1.09	6	+ 0.075	− 0.045	+ 0.33	2	+ 0.075	+ 0.048	− 0.71
D_1	—	—	—	—	8	+ 0.043	− 0.051	+ 1.79	2	+ 0.053	− 0.042	− 1.51
D_2	—	—	—	—	3	+ 0.032	− 0.113	+ 1.67	2	+ 0.043	+ 0.051	+ 8.38
D_3	1	+ 0.006	− 0.094	− 2.89	5	− 0.032	− 0.124	+ 2.19	1	+ 0.024	− 0.090	− 8.22
D_4	2	− 0.011	− 0 003	+ 0.32	8	+ 0.001	− 0.022	+ 3 51	1	− 0.020	+ 0.040	+ 7.46
D_5	6	− 0.026	− 0.005	+ 1.78	6	+ 0.017	− 0.020	+ 4.80	1	− 0.020	− 0.009	+ 3.47
D_6	1	+ 0.034	− 0.039	− 0.87	6	− 0.138	+ 0.061	+ 2.91	—	—	—	—
D_7	2	+ 0.034	− 0.056	− 1.34	3	− 0.089	− 0.006	− 2.61	2	− 0.062	− 0.035	+ 2.71
D_8	—	—	—	—	9	+ 0.005	− 0.041	+ 0.17	1	− 0.076	− 0.055	− 1.85
D_9	2	+ 0.008	− 0.058	− 0.79	4	− 0.002	− 0.047	− 2 35	2	− 0.028	− 0.093	− 3.49
D_{10}	2	+ 0.016	+ 0.011	− 2.85	10	+ 0.001	− 0.097	− 1.87	1	+ 0.036	+ 0.010	+ 1.76
D_{11}	5	+ 0.040	+ 0.003	− 2.40	5	+ 0.067	− 0.007	− 1.05	3	− 0.015	− 0.035	+ 0.57
D_{12}	3	+ 0.032	− 0.003	+ 1.29	8	+ 0.072	− 0.050	− 0.33	2	+ 0.023	+ 0.001	− 3.20
E_1	1	+ 0.105	+ 0.011	+ 2.79	5	+ 0.028	− 0.059	− 0.53	1	− 0.092	− 0.092	+ 0.77
E_2	1	+ 0.037	+ 0.059	+ 7.47	11	+ 0.037	− 0.018	+ 3.44	2	+ 0.002	+ 0.002	− 1.43
E_3	2	− 0.018	− 0.014	+ 1.80	8	+ 0.046	− 0.068	+ 3.64	2	− 0.018	+ 0.056	+ 4.96
E_4	4	− 0.014	− 0.002	+ 0.85	17	− 0.018	− 0.025	+ 3.57	—	—	—	—
E_5	7	− 0.035	− 0.023	+ 1.19	11	− 0.013	− 0.018	+ 1.78	—	—	—	—
E_6	3	− 0.137	− 0.001	+ 0.89	9	− 0.058	− 0.037	− 1.12	2	− 0.014	− 0.164	+ 5.72
E_7	2	+ 0.016	− 0.013	− 0.61	10	− 0.067	− 0.072	− 4.06	1	+ 0.026	− 0 017	− 1.36
E_8	2	+ 0.018	− 0.032	+ 0.70	11	+ 0.008	− 0.076	− 1.10	2	− 0.078	− 0.098	− 0.37
E_9	1	+ 0.008	+ 0.004	+ 3.72	5	+ 0.035	+ 0.001	− 1.27	1	− 0.024	− 0.001	+ 6.91
E_{10}	3	+ 0 003	+ 0.009	− 0.16	7	+ 0.013	− 0.042	+ 3.50	3	+ 0.043	− 0.017	− 0.81
F_1	1	− 0.003	+ 0.011	+ 4.22	7	− 0.021	+ 0.040	+ 2.04	2	− 0.012	+ 0.084	− 1.82
F_2	1	+ 0.146	+ 0.174	+ 1.58	10	− 0.061	− 0.065	+ 0.65	2	− 0.001	− 0.137	− 1.66

TABLE XVII.

Square	magn $\leq$ 4.9			
	n	x_0	y_0	z_0
A_1	9	+ 0.0063	— 0.0229	— 1.938
A_2	16	— 0.0075	+ 0.0215	— 2.718
B_1	23	+ 0.0478	— 0.0375	— 0.819
B_2	26	+ 0.0395	— 0.0289	+ 1.176
B_3	18	+ 0.0144	— 0.0674	+ 0.409
B_4	18	— 0.0758	— 0.0622	+ 1.555
B_5	18	— 0.0554	— 0.0443	— 0.570
B_6	17	— 0.0313	— 0.0218	— 2.446
B_7	20	— 0.0490	+ 0.0336	— 2.620
B_8	23	+ 0.0262	+ 0.0281	— 4.590
B_9	26	+ 0.0155	+ 0.0113	— 3.615
B^{10}	25	+ 0.0454	— 0.0197	— 2.558
C_1	20	+ 0.0234	— 0.0405	+ 1.371
C_2	21	+ 0.0337	— 0.0652	+ 0.496
C_3	38	+ 0.0388	— 0.0366	+ 5.011
C_4	18	— 0.0191	— 0.0746	+ 2.848
C_5	16	— 0.0709	— 0.0534	+ 4.186
C_9	14	— 0.0433	— 0.0671	+ 0.404
C_7	13	— 0.1394	— 0.0365	— 1.994
C_8	20	— 0.0263	— 0.0297	+ 3.601
C_9	15	— 0.0454	— 0.0094	— 5.261
C_{10}	27	+ 0.0309	— 0.0434	— 4.229
G_{11}	20	+ 0.0315	— 0.0640	— 1.998
C_{12}	23	+ 0.0768	— 0.0691	— 0.052
D_1	12	+ 0.0249	— 0.0472	+ 1.080
D_2	11	+ 0.0408	— 0.0281	+ 2.744
D_3	29	— 0.0144	— 0.0371	+ 2.547
D_4	31	— 0.0049	— 0.0007	+ 5.200
D_5	18	— 0.0032	— 0.0158	+ 3.309
D_6	10	— 0.0934	+ 0,0236	+ 2.480
D_7	13	— 0.0761	— 0.0313	— 0.458
D_8	19	— 0.0225	— 0.0826	— 1.156
D_9	29	+ 0.0003	— 0.0545	— 2.104
D_{10}	22	+ 0.0082	— 0.0507	— 2.392
D_{11}	22	+ 0.0403	— 0.0236	— 1.490
D_{12}	17	+ 0.0586	— 0.0239	+ 0.431
E_1	12	+ 0.0483	— 0.0379	+ 0.380
E_2	21	+ 0.0386	+ 0.0171	+ 3.148
E_3	24	+ 0.0021	— 0.0091	+ 3.308
E_4	44	— 0.0244	— 0.0151	+ 3.352
E_5	35	— 0.0321	— 0.0102	+ 1.526
E_6	42	— 0.0615	— 0.0386	+ 0.770
E_8	28	— 0.0475	— 0.0610	— 1.817
E_8	32	+ 0.0004	— 0.0743	— 0.952
E_9	15	+ 0.0382	— 0.0291	+ 0.459
E_{10}	21	+ 0.0379	— 0.0320	+ 1.728
F_1	20	—0. 0069	+ 0.0439	+ 1.491
F_2	17	—0. 0294	— 0.0492	+ 0.872

TABLE XVIII.

The parallaxes of stars brighter than magn 4.9.

	Spectral type	Number of stars	$\vartheta_1 = \pi$	Results of Campbell all magnitudes			Results of Kapteyn computed for magn. 5.0	
				Number	π	Remarks	Number	π
magn $\leq$ 4.9	*B*	197	0″.0075 ± 0″.0003	312	0″.0061	*B*0—*B*5	440	0″.0068
				90	0.0129	*B*8—*B*9		
	A	192	0.0162 ± 0.0010	172	0.0166		1088	0.0098
	F	128	0.0270 ± 0.0031	180	0.0354			
	G	107	0.0127 ± 0.0016	118	0.0223		1036	0.0224
	K	338	0.0176 ± 0.0012	346	0.0146			
	M	63	0.0144 ± 0.0023	71	0.0106		101	0.0111
	All spectral types	1028	0.0163 ± 0.0005					

In the table XVIII the values of ϑ_1 are given for each spectral class. It may be observed that, when using the units Siriometer and Stellar year in connection with angular measure, the values of ϑ_1 will be equal to the parallaxes.

For a comparison with other results, I give the values of the parallaxes derived by Campbell and Kapteyn. None of these results, however, are comparable with the present ones, since here the stars are limited to the magnitude 5.0. On the other hand Campbell has determined the parallaxes in a special way. The average values of the components of the proper motion after a reduction for the sun's velocity he directly compares with the average radial velocity and thus derives the parallaxes. This method, only applicable when the stars in each spectral class are at the same distance, is surely a rough approximation.

The agreement between the present results and those of Campbell seems to be satisfactory with the exception of the parallaxes of the *F* and *G* class stars. It must however be noted that Campbell only rejected very few stars, whose proper motions were extremely large [1].

52. In order to examine whether the parallax is a function of galactic latitude I have for the group magnitude ≤ 4.9 in the way described on page 19 divided the sky into two groups of squares surrounding the plane and the pole of the

[1] See Lick Obs. Bulletin VI page 131.

Milky Way. In the computations we have however to consider the different relative velocities of the sun table IVb page 20. In this table the value of S is for the stars surrounding the pole of the Milky Way very small — 3.864 Sm. —

There is, however, no reason to assume the value of S to vary in relation to stars of different galactic latitudes. On the other hand the apex solution from stars surrounding the pole of the Milky Way will give a rather uncertain value of S on account of the position of the apex near the galactic plane. This velocity must however necessarily exceed the lowest velocity observed for the different spectral class stars. I have therefore in the solution used the values 4.000 and 4.100.

The following table gives the results:

Parallaxes of stars of magn. ≤ 4.9:

Surrounding the galactic plane		Surrounding the galactic pole		
n	π	n	π	
611	$0''.0135$	417	$0''.0175$	(assuming $S = 4.100$)
		»	$0''.0181$	(» $S = 4.000$)

The large difference in these parallaxes is very striking. The value $\pi = 0.0181$ for stars in the polar zone I have assumed as the most probable; it is rather too small than too large.

Is the difference in the parallaxes caused by the distribution in the two zones of the stars, especially the class B stars, whose parallax is very small? If this is the case we may from the parallax values in table XVIII compute the parallaxes in question. Putting the weights equal to the number of the stars this calculation gives:

Computed parallaxes of the stars of magn. ≤ 4.9:

Surrounding the galactic plane	Surrounding the galactic pole
$\pi = 0''.0154$	$\pi = 0''.0165$

The variation is not sufficient to establish the first results to be exclusively caused by the distribution of stars of different spectral classes in relation to the Milky Way.

53. Assuming the mean parallax of stars of magn. ≤ 4.9 to be expressed as a function of galactic latitude through the formula

$$\vartheta_1 = a + b \sin \beta,$$

an integration of this function for the two zones $-30^0 < \beta < +30^0$ and $\pm 60^0 < \beta < \pm 90^0$ will give the numerical value of the coefficients a and b. Thus the variation of ϑ_1 with galactic latitudes is approximatively given by

$$\vartheta_1 = 0''.0134 + 0''.0058 \sin \beta.$$

54. In order to get a control of the determinations of the parallaxes in the table XIX I made from the equation (50) a separation of the observations in proper motion. From these results I determined the values of $\vartheta_1 S$ as well as the positions of the apices. The following table contains the results.

TABLE XIX.

The apex solution from the proper motions of stars brighter than magn. 4.9 [1].

Class	$\vartheta_1 U_0''$	$\vartheta_1 V_0''$	$\vartheta_1 W_0''$	$\vartheta_1 S$	Apex		$\vartheta_1 S : S$
					α	δ	
B	− 0″.00132	+ 0″.03080	− 0″.01932	0″.03647	272.°3	+ 32°.0	0″.0078
A	+ 0.00212	+ 0.05903	− 0.02919	0.06588	267.9	+ 26.3	0.0158
F	+ 0.01366	+ 0.10244	− 0.06668	0.12300	262.5	+ 32.8	0.0299
G	+ 0.00506	+ 0.03555	− 0.03759	0.05199	261.9	+ 46.3	0.0131
K	− 0.00842	+ 0.04934	− 0.05258	0.07259	279.7	+ 46.3	0.0176
M	− 0.00444	+ 0.05837	− 0.03333	0.06736	274.4	+ 29.7	0.0152
Magn. $\leq$4.9	− 0.00117	+ 0.05428	− 0.03937	0.06706	271.2	+ 35.9	0.0163

There is a large difference in the value of the declination of the apex for stars brighter than magn. 5.0 when determined from radial velocities and proper motions. The former determination gave (see table IV a) for the apex $\alpha = 271°.5$, $\delta = + 26°.3$. The mean error, about two or three degrees, is not sufficient to explain the difference, which no doubt is in some way caused by the systematic motions of the stars. It is also probable that the combined treatment of radial velocities and proper motions is a rather complicate problem, although the solutions from the two sorts ofobservations point approximatively in the same direction

[1] In this table as well as in the tables IV a, IV b, containing the solar velocity results, I have like other authors, defined the components of the sun's relative velocity in such a way that the velocity is considered as negative in relation to the point in the sky, the apex, towards which the sun is moving. However, for the solar velocity — S — no signs are given in the tables.

CHAPTER V.

The moments of the second order from p oper motion and radial velocity observations.

55. In this chapter I will make a short examination of the moments of the second order.

When examining the velocity distribution as derived from proper motions, we have to consider the formulae which express the angular mean velocities in linear measure [1].

$$
\begin{aligned}
\vartheta_1^{\,2} N_{200} &= \nu_{200}\, q' - (1 - q')\, x_0^{\,2}, \\
\vartheta_1^{\,2} N_{020} &= \nu_{020}\, q' - (1 - q')\, y_0^{\,2}, \\
\vartheta_1^{\,2} N_{110} &= \nu_{110}\, q' - (1 - q')\, x_0\, y_0 .
\end{aligned} \tag{51}
$$

Here q' is the fundamental constant in the star density function, whose numerical value we are now going to determine.

56. At first we may from the proper motion observations regard the velocity distribution as expressed through the ellipsoidal hypothesis.

For this purpose we have to determine from the formulæ (51) the parameters in the expressions (30) page 33 which characterize the ellipsoidal surface [2].

It is, however, not necessary to treat the expressions (51) in their present form. Using in the calculations the observed moments

$$\nu_{200}, \nu_{020}, \nu_{110}$$

we get a determination of the velocity distribution of the so called »apparent cross-motions».

The values of the moments ν_{200}, ν_{020}, ν_{110} as well as of ν_{002}, computed about the mean observed, are tabulated in the table XX.

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[1] See CHARLIER cited memoir.

[2] The solution from the first two formulae of (51) I have given in meddelande No 59. The complete solution, however, including also the third equation of (51) is deduced by S. WICKSELL and published by him in Meddelande No 60.

TABLE XX.

The moments of the second order of the observed proper motions and radial velocities.

Square	ν_{200}	ν_{020}	ν_{002}	ν_{110}
A_1	+ 0.00609	+ 0.01391	+ 11.07	+ 0.00701
A_2	+ 0.00305	+ 0.00519	+ 13.59	— 0.00006
B_1	+ 0.01164	+ 0.00449	+ 7.98	+ 0.00377
B_2	+ 0.00543	+ 0.00437	+ 6.86	+ 0.00338
B_3	+ 0.00190	+ 0.00827	+ 12.49	— 0.00130
B_4	+ 0.01484	+ 0.00723	+ 12.97	+ 0.00574
B_5	+ 0.00915	+ 0.00401	+ 11.64	— 0.00163
B_6	+ 0.01187	+ 0.00957	+ 5.08	+ 0.00152
B_7	+ 0.01450	+ 0.01809	+ 10.12	+ 0.00215
B_8	+ 0.00427	+ 0.00585	+ 14.55	+ 0.00038
B_9	+ 0.00277	+ 0.00563	+ 7.47	+ 0.00271
B_{10}	+ 0.00652	+ 0.00513	+ 6.48	+ 0.00089
C_1	+ 0.00920	+ 0.00298	+ 9.53	+ 0.00130
C_2	+ 0.01157	+ 0.01378	+ 15.85	+ 0.00295
C_3	+ 0.00734	+ 0.00265	+ 13.38	+ 0.00017
C_4	+ 0.00180	+ 0.00727	+ 20.15	+ 0.00059
C_5	+ 0.00456	+ 0.00208	+ 13.27	— 0.00057
C_6	+ 0.02237	+ 0.00380	+ 15.48	— 0.00200
C_7	+ 0.02384	+ 0.00877	+ 9.16	— 0.00240
C_8	+ 0.01013	+ 0.00405	+ 24.49	— 0.00039
C_9	+ 0.00649	+ 0.00767	+ 14.22	+ 0.00320
C_{10}	+ 0.00318	+ 0.00811	+ 12.76	— 0.00115
C_{11}	+ 0.00596	+ 0.00605	+ 10 31	— 0.00315
C_{12}	+ 0 01703	+ 0.02064	+ 11.51	— 0.00555

57. If we use the form (51) for the linear moments in a certain square, it is easily found from the usual transformation of coordinates that the expression for the linear moments in another system, say the galactic system — G —, will have the same form as (51) so that in this system

$$
\begin{aligned}
\vartheta_1^{\,2} N^G_{200} &= \nu^G_{200}\, q' - (1 - q')\, \xi_0^{\,2} \\
\vartheta_1^{\,2} N^G_{020} &= \nu^G_{020}\, q' - (1 - q')\, \eta_0^{\,2} \\
\vartheta_1^{\,2} N^G_{002} &= \nu^G_{002}\, q' - (1 - q')\, \zeta_0^{\,2}
\end{aligned} \tag{52}
$$

and similar equations for the correlation moments. Here ξ_0, η_0 and ζ_0 denote the components of the sun's relative velocity in the system G.

The moments ν^G_{ijk} are the »apparent moments» in relation to the same system and may be computed from the apparent moments in the squares.

From the solution of the velocity ellipsoid of the apparent proper motions, the following three axes and directions were found:

TABLE XXI.

The distribution of the apparent velocities (proper motion) of stars of magn ≤ 4.9.

Axes directed towards the	Mean velocity	Position	
		α	δ
Principal vertex	0″.1125	284°.1	− 8°.0,
Pole of the Milky Way...	0″.0724	180°.0	+ 8°.2,
Third direction..	0″.0826	235°.0	+ 78°.1.

The velocity ellipsoid has, as shown from these results a distinct principal direction in agreement with the radial velocity results, but is in contrast with this flattened in the plane of the Milky Way. In table XXI the unit used is seconds of arc per year.

58. In the equation (52) we know the values of ϑ_1, as well as the values of N^{σ}_{200}, N^{σ}_{020} and N^{σ}_{002} (table XIII page 42). The velocity components ξ_0, η_0, ζ_0 of the sun, relative to the stars we may compute from the known solar velocity and the apex direction.

The axes of the apparent velocity ellipsoids are further very nearly coincident with the axes of the galactic system of coordinates — G — and we may in an approximative computation for the moments ν^{σ}_{ijk} in the formulae (52) substitute the squares of the three mean velocities in table XXI.

From the three equations (52) we now get the values of q'. The following results were obtained:

Axis directed towards:	corresponding value of q':
Principal vertex	+ 0.466
Pole of the Milky Way	+ 0.690
Third direction	+ 0.403

The values of q' obtained are unexpectedly small, with the exception of the result from the comparison between the mean velocities perpendicular to the plane of the Milky Way. The mean velocity in this direction is, however, from the proper motion solution, almost exclusively determined from stars surrounding the plane of the Milky Way. Introducing into the formulae the corresponding mean value of ϑ_1, — 0.0135 — for these stars, the value of q' will diminish from + 0.690 to + 0.493.

59. It is however possible in another way to get an approximative value of q'.

The proper motions in a zone surrounding the galactic pole may be quite comparable with the radial motion in the galactic plane. The proper motion components in the direction of the pole of the Milky Way of stars situated in the galactic plane are on the other hand comparable with the radial velocities of stars in the polar zone.

In this way I have derived the following results noted in the table below. The first columns indicate the moments compared, and the value of ϑ_1 used in the formulae. The last column gives the corresponding value of q'. The next four tabulated determinations were made from a comparison between the moments over the whole sky.

Compared moment of the velocities of the stars surrounding:

the galactic plane, (Mean $\pi = 0.0135$		the galactic pole, (Mean $\pi = 0.0181$)	Corresponding value of q'
N_{002}	compared with	N_{020}	$+ 0.511$
N_{002}	»	N_{200}	$+ 0.409$
N_{200}	»	N_{002}	$+ 0.542$
N_{020}	»	N_{002}	$+ 0.530$

Comparison over the whole sky (difference of parallaxes considered)

				corresponding value of q'
between	N_{200}	and	N_{002}	$+ 0.467$
»	N_{020}	»	N_{002}	$+ 0.467$

Comparison over the whole sky (parallaxes adopted $= 0.0163$.)

between	N_{200}	and	N_{002}	$+ 0.493$
»	N_{020}	»	N_{002}	$+ 0.446$

There seems to be no doubt that a mean value of q' equal to $+ 0.50$ derived from the first four equations will statisfy the equation (52). However, the numerical values of q' obtained in the different ways are probably too small, but on the other hand a variation of the mean parallax, or a variation of the sun's relative velocity will largely influence the results.

Indeed it was shown, first by Charlier and then by Wicksell, that a value of q' so small as $+ 0.50$ will not suit the moments of the third and fourth orders, as derived from proper motion observations, which moments for the value $q' = 0.50$ are very much diverging from zero. The relative velocity of the sun is then assumed to be 20 km per second.

For the rest I refer to the authors cited.

Summary.

I. The magnitude of the sun's relative velocity seems to be highly dependent on the spectral class, and shows very high values for the spectral classes B and M. In the same manner the constant K is a function of spectral class, and the numerical value of this constant seems to be closely correlated with the solar velocity.

As for the radial velocity observations used here, the value of K seems, especially for stars of types B and M, in some way to be dependent on the angular distances of the stars from the principal vertex $\alpha = 274^{0},6$, $\delta = -12^{0},0$ and the antivertex. In these regions the value of K is larger than for other parts of the sky.

2. As for the velocity distribution, the generalized ellipsoidal hypothesis was applied. It was then found that, when the stars were divided into three groups according to their spectral classes, as well as when all stars were treated together, the velocity surfaces were represented by three axial ellipsoids with distinct extension in the direction of the principal vertex, but flattened perpendicular to the plane of the Milky Way. The excentricity of the ellipsoid was found to be largest for stars of the so called earlier types and smallest for stars of the so called later types. For the group containing stars of magn $\leq 4,9$, the position of the ellipsoid was closely correlated with the galactic system.

A second solution, where the spectral classes B, A, F, G, K, M were treated separately, led to the following results:

In contradistinction to the other stars, the class B stars have their velocity distribution represented by an ellipsoid of revolution flattened in the plane of the Milky Way. No principal vertex direction seems to exist for these stars.

For the stars of class A the principal vertex direction is very distinct; further the velocity ellipsoid is flattened perpendicular to the plane of the Milky Way. Indeed the class A stars show, compared with the other spectral classes, the most excentric velocity surface.

The other spectral class stars have their velocity distribution represented by three axial ellipsoids with the same position as for the class A stars. Excepting the class B stars, the mean velocity of the stars in the direction of the principal vertex seems to be almost constant, while the mean velocities in the other directions are slowly increasing when passing from the class A to the class M. For stars of this latter class the velocity distribution seems to be quite spherical.

3. Concerning the moments of higher order for stars brighter than 5,0 the characteristics of the third and fourth orders in the three main directions in the galactic system were found to be very small. However, a distinct negative skewness was found in the direction of the principal vertex, and, furthermore, a positive excess in the other two directions.

4. A comparison between proper motion and radial velocity of the stars, the mean parallaxes were derived for the different spectral classes including stars of magn $\leq 4{,}9$.

5. A comparison between the moments of the second order from proper motion and radial velocity observations led to the unexpected small value $+0.50$ of the constant q'. This value is, however, strongly influenced by the parallax assumed.

Observatory Lund jan. 1915.

In concluding I desire to express to Prof. C. V. L. Charlier my hearty thanks for the interest he has taken in my work and for the valuable advice and directions he has given me during its progress. I also take this opportunity of thanking my colleague, Mr S. D. Wicksell for pleasant and profitable collaboration; our common discussions on stellar astronomical problems have, especially in the case of the present paper, been of extremely great use to me.

To the Royal Physiographic Society I am indebted for monetary assistance.

Errata:

In the table X page 30 the lines indicated by K and M are to be exchanged.

Summary.

I. The magnitude of the sun's relative velocity seems to be highly dependent on the spectral class, and shows very high values for the spectral classes *B* and *M*. In the same manner the constant *K* is a function of spectral class, and the numerical value of this constant seems to be closely correlated with the solar velocity.

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A second solution, where the spectral classes *B*, *A*, *F*, *G*, *K*, *M* were treated separately, led to the following results:

In contradistinction to the other stars, the class *B* stars have their velocity distribution represented by an ellipsoid of revolution flattened in the plane of the Milky Way. No principal vertex direction seems to exist for these stars.

For the stars of class *A* the principal vertex direction is very distinct; further the velocity ellipsoid is flattened perpendicular to the plane of the Milky Way. Indeed the class *A* stars show, compared with the other spectral classes, the most excentric velocity surface.

The other spectral class stars have their velocity distribution represented by three axial ellipsoids with the same position as for the class *A* stars. Excepting the class *B* stars, the mean velocity of the stars in the direction of the principal vertex seems to be almost constant, while the mean velocities in the other directions are slowly increasing when passing from the class *A* to the class *M*. For stars of this latter class the velocity distribution seems to be quite spherical.

3. Concerning the moments of higher order for stars brighter than 5,0 the characteristics of the third and fourth orders in the three main directions in the galactic system were found to be very small. However, a distinct negative skewness was found in the direction of the principal vertex, and, furthermore, a positive excess in the other two directions.

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